რეოლოგიური ეფექტები გრანულარულ ასტროფიზიკულ დისკებში

Rheology of granular astrophysical discs

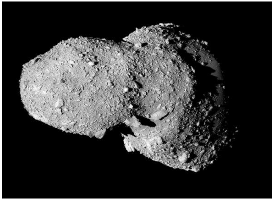
ლუკა პონიატოვსკი

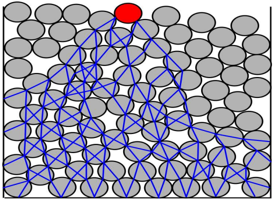
[1]Abastumani Astrophysical Observatory

3 მარ. 2017 წ.

Granular Flow

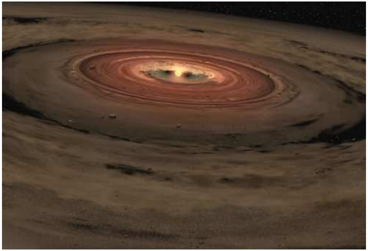
- Size range $1cm$ up to $400AU$

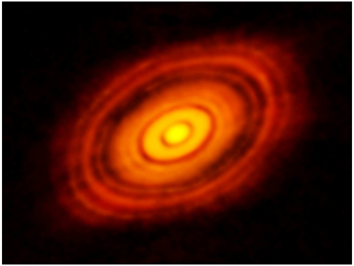


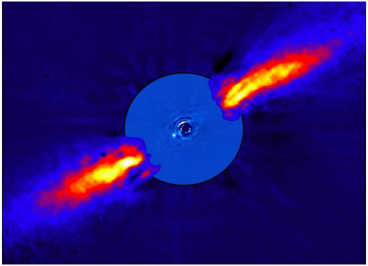


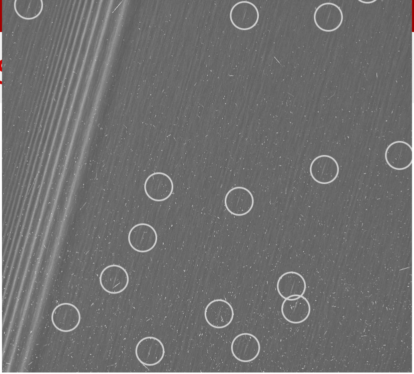
Granular Disc Flows

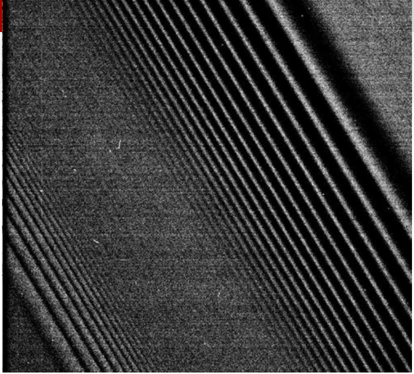
1. Protoplanetary discs
2. Debris discs
3. Planetary rings
4. Exoplantary rings

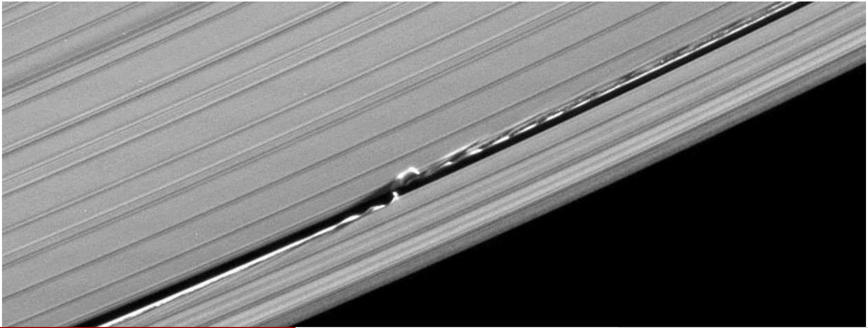












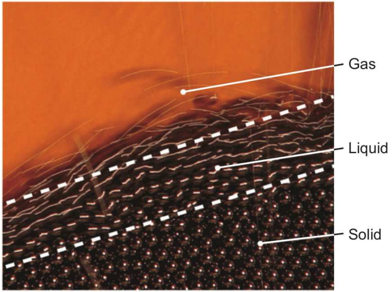
Rheological Flow

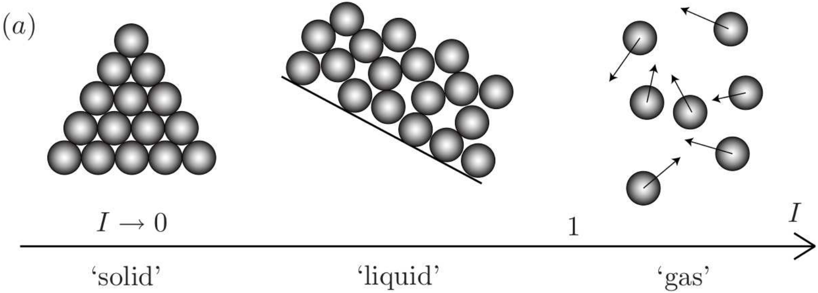
$$\nu = \nu(\mu, P, \mathbf{V})$$

- Pressure thickening
- Pressure thinning
- Shear thickening
- Shear thinning

Physical Description

1. Kinetic (Gaseous)
2. Liquid (Fluid)
3. Soft matter (Solid)

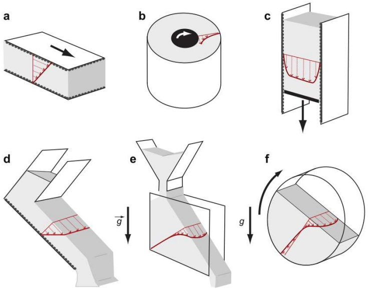


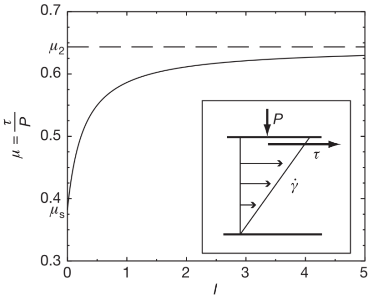


Granular Phenomenology

- Depends on Filling factor Φ and restitution coefficient e.
- Sensible to Shear stress

$$\mu = \mu(I) \quad \text{Friction Coeff. ,} \quad I = \frac{t_{micro}}{t_{macro}} \quad \text{Inertial Number}$$

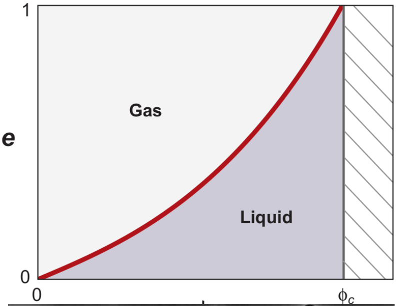


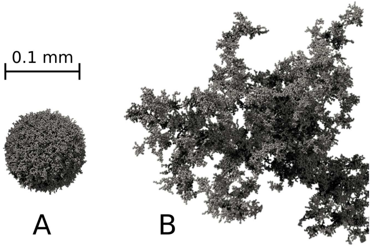


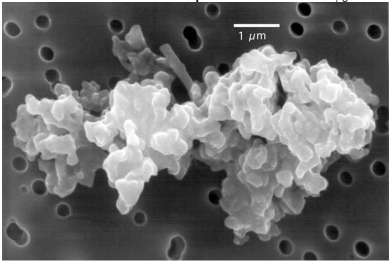
Granular Flow in Space

Restitution Coeff.: $e = \frac{v_2}{v_1}$

Filling factor: $\phi = \frac{V[Grains]}{V[Total]}$







Rotating Fluid

Navier-Stokes equation:

$$\rho \left\{ \frac{\partial}{\partial t} + v_k \frac{\partial}{\partial x_k} \right\} v_i = -\frac{\partial P}{\partial x_i} + \frac{\partial \Phi}{\partial x_i} + \frac{\partial \tau_{ik}}{\partial x_k}$$

Viscous stress tensor:

$$\tau_{ij} = \eta \dot{\gamma}_{ij} \ ,$$

Strain tensor:

$$\dot{\gamma}_{ik} = \frac{\partial V_k}{\partial x_i} + \frac{\partial V_i}{\partial x_k} \ ,$$

Newtonian Fluid: $\eta = const$

Non-Newtonian Fluid: $\eta \neq const$

Rotating Fluid

Cylindrical Co-ordinates:

$$\frac{\partial v_\phi}{\partial t} + \left(v_r \frac{\partial}{\partial r} + \frac{v_\phi}{r}\frac{\partial}{\partial \phi} + v_z \frac{\partial}{\partial z} \right) v_\phi + \frac{v_r v_\phi}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \phi} + \frac{1}{r}\frac{\partial \Phi}{\partial \phi} +$$

$$+ \frac{\eta}{\rho}\left(\Delta v_\phi - \frac{v_\phi}{r^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \phi} \right) + \frac{1}{\rho}\frac{\partial \eta}{\partial r}\left(\frac{1}{r}\frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r} \right) +$$

$$+ \frac{2}{r}\frac{\partial \eta}{\partial \phi}\left(\frac{1}{r}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right) + \frac{1}{\rho}\frac{\partial \eta}{\partial z}\left(\frac{\partial v_\phi}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \phi} \right)$$

Accretion Discs

α model (Shakura and Sunyaev 1973):
turbulent viscosity

$$\nu_{\text{eff}} = \alpha H C_s$$

$$\nu_{\text{eff}} \gg \nu$$

$\nu_{\text{eff}} \neq const$:

- Viscous instability
  (Lightman and Eardley 1974, Shakura and Sunyaev 1976)
- Viscous overstability
  (Kato 1978, Blumenthal at. el. 1984)

Viscous Instability

General assumption: $\nu \propto \sigma^{\beta}$ (Ward 1981)

$$\frac{\mathrm{d}(\nu\sigma)}{\mathrm{d}\sigma} < 0$$

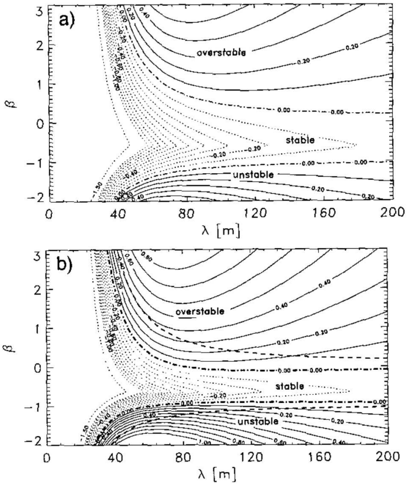
σ-surface density (Opacity)

- Necessary criterion $\beta < -1$
- No shear effects considered
- Unlike to occur

Viscous Overstability

$\beta > \beta_{critical}$ pulsational instability appears

$$\alpha^3 + \left(\xi + \frac{7}{3}\nu\right) k^2\alpha^2 + \left(\Omega^2 - 2\pi G\sigma_0 k + c^2k^2 + \left(\xi + \frac{4}{3}\nu\right)\nu k^4\right)\alpha - (2\pi G\sigma_0 k - c^2k^2 - 3(\beta + 1)\Omega^2)\nu k^2 = 0. \quad (28)$$



- Compressible mechanism
- $\beta_{critical} = 2$

(Schmit and Tscharnuter 1995[Fig. 1])

Local description

- Experimental studies of granular flows:
  1. Friction coefficient $\mu(I)$
  2. Pressure P
  3. Second invariant of the strain rate ξ

$$\eta = \frac{\mu(I)P}{\xi}\ , \qquad \xi = \sqrt{\frac{1}{2}\dot{\gamma}_{ij}\dot{\gamma}_{ij}}\ ,$$

- Possible to write local constitutive equation:

(Forterre & Pouliquen 2008 , Jop et. al. 2006).

Equilibrium Solutions

Axisymetric cylindrical equilibrium :

$$\rho_0, P_0 = const$$
$$\eta_0 = \eta_0(r)$$
$$\mathbf{v}_0 = (0, v_{0\phi}, 0)$$

$$r\Omega^2 = -\frac{\partial \Phi}{\partial r}$$
$$\left(r\frac{\partial^2 \Omega}{\partial r^2} + 3\frac{\partial \Omega}{\partial r} \right) \eta_0 + r\frac{\partial \Omega}{\partial r}\frac{\partial \eta_0}{\partial r} = 0$$

$$\Omega \propto r^{-q}$$
$$\eta_0 \propto r^{q-2}$$

- Keplerian rotation $q = -3/2$
- Rayleigh stability criterion: $(r^2\Omega(r))' > 0$

$$\frac{\partial \eta_0}{\partial r} < 0$$

Local Linear Analysis

- Linear Perturbations:

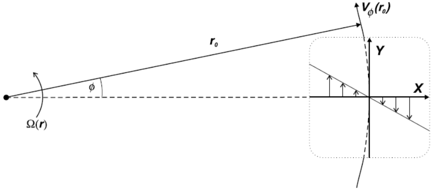
$$P = P_0 + P' \ , \qquad \eta = \eta_0 + \eta'(r) \ , \qquad \mathbf{v} = \mathbf{v}_0 + \mathbf{v}'$$

$$\frac{\mathrm{D}v'_\phi}{\mathrm{D}t} + \left(2\Omega + r\frac{\partial \Omega}{\partial r}\right) v'_r = -\frac{1}{\rho r}\frac{\partial P'}{\partial \phi} + \nu_0 \left(\Delta v'_\phi - \frac{v'_\phi}{r^2} + \frac{2}{r^2}\frac{\partial v'_r}{\partial \phi}\right) +$$

$$+ \frac{\eta'}{\rho}\left(r\frac{\partial^2 \Omega}{\partial r^2} + 3\frac{\partial \Omega}{\partial r}\right) + \frac{1}{\rho}\frac{\partial \eta_0}{\partial r}\left(\frac{\partial v'_\phi}{\partial r} - \frac{v'_\phi}{r} + \frac{1}{r}\frac{\partial v'_r}{\partial \phi}\right) + \frac{r}{\rho}\frac{\partial \eta'}{\partial r}\frac{\partial \Omega}{\partial r}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial \phi}$$

Local Co-rotating System



$$\begin{pmatrix} x \equiv r - r_0 \\ y \equiv r_0(\phi - \Omega(r_0)t0) \\ t \equiv t \end{pmatrix}$$

Local approximation means:

1. system characteristic length is bigger than the length-scale of main gradients. (e.i. $\lambda^2 \ll L^2$)
2. characteristic length of rheology is smaller than given fiducial radius. (e.i. $\lambda_{visc} \ll \lambda$)

$$\frac{\mathrm{D}v'_y}{\mathrm{D}t} - 2\mathrm{B}v_x = -\frac{1}{\rho_0}\frac{\partial P'}{\partial y} + \nu_0 \Delta_{xyz} v'_y + \frac{\mathrm{A}}{\rho_0 r_0}\eta' +$$

$$+ \frac{\nu_0}{2r_0}\left(\frac{\partial v'_y}{\partial x} + \frac{\partial v'_x}{\partial y}\right) + \frac{2\mathrm{A}}{\rho_0}\frac{\partial \eta'}{\partial x}$$

Local Rheology

- General for of the local constitutive law:

$$\eta = \eta(P, \xi)$$

$$G_P \equiv \left(\frac{\partial \eta}{\partial P}\right)_\xi \,, \qquad \text{Pressure rheology parameter}$$

$$G_S \equiv \frac{1}{\rho}\left(\frac{\partial \eta}{\partial \xi}\right)_P \,, \qquad \text{Shear rheology parameter}$$

$$\frac{\eta'}{\rho} = G_P \frac{P'}{\rho} + G_S \left(\frac{\partial V'_x}{\partial y} + \frac{\partial V'_y}{\partial x}\right)$$

Kelvin Modes

- Incomprehensible perturbations

$$\begin{pmatrix} \mathbf{V}' \\ P'/\rho_0 \\ \eta'/\rho_0 \end{pmatrix} \propto \begin{pmatrix} \mathbf{u} \\ -ip \\ -i\bar{\nu} \end{pmatrix} \times e^{i\mathbf{rk(t)}} \qquad k_x(t) = k_x(0) + 2Ak_y t$$

$$\begin{aligned}
\dot{u}_x(t) &= 2\Omega_0 u_y(t) - k_x(t)p(t) - \nu k^2(t)u_x(t) + 2Ak_y\bar{\nu}(t) \ , \\
\dot{u}_y(t) &= 2Bu_x(t) - k_y p(t) - \nu k^2(t)u_y(t) + 2Ak_x(t)\bar{\nu}(t) \ , \\
\dot{u}_z(t) &= -k_z p(t) - \nu k^2(t)u_z(t) \ , \\
0 &= k_x(t)u_x(t) + k_y u_y(t) + k_z u_z(t) \ , \\
\bar{\nu}(t) &= G_P p(t) - G_S(k_x(t)u_y(t) + k_y u_x(t)) \ ,
\end{aligned}$$

- Initial value problem

Stability Analysis

$\sim \exp(-i\omega(t)t)$:

$$\omega = \pm \left(\bar{\kappa}^2 - W^2\right)^{1/2} + \mathrm{i}\left(W - \nu k^2\right) \ ,$$

$$\bar{\kappa}^2 = \left(-4B\Omega - 4A^2 G_S k_x k_y\right) \frac{k_z^2}{k^2 - 4AG_P k_x k_y} \ ,$$

Rheological modification of epicyclic frequency.

$$\begin{array}{lll} \bar{\kappa}^2 > W^2, & W > \nu k^2 & : \text{overstability} \\ \bar{\kappa}^2 < W^2, & W + \sqrt{W^2 - \bar{\kappa}^2} > \nu k^2 & : \text{instability} \end{array}$$

Stability Analysis

$$W = \sigma_A + \sigma_P + \sigma_S \ ,$$

$$\sigma_A = \frac{Ak_x k_y}{k^2 - 4AG_P k_x k_y} \ , \qquad \text{transient growth}$$

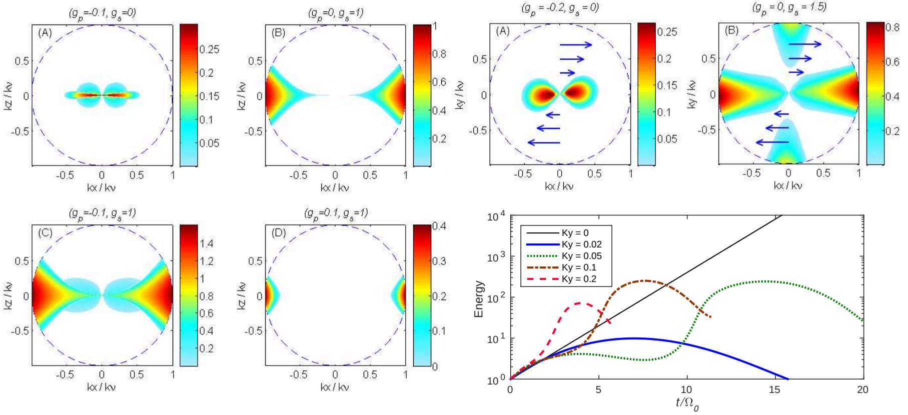
$$\sigma_P = 2AG_P \frac{(\Omega k_x^2 + Bk_y^2)}{k^2 - 4AG_P k_x k_y} \ , \qquad \text{pressure effect}$$

$$\sigma_S = -AG_S \frac{(k_x^2 - k_y^2)^2 + k_\perp^2 k_z^2}{k^2 - 4AG_P k_x k_y} \ , \qquad \text{shear (strain) effect}$$

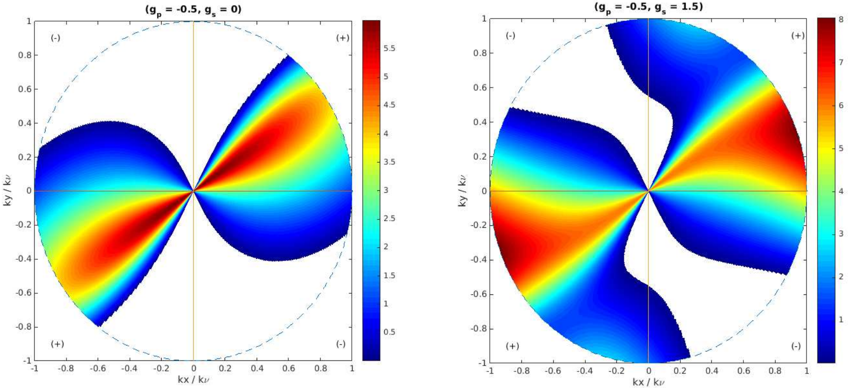
Axisymetric limit: $k_y = 0$ $\quad \sigma_A = 0$

$\left| \frac{k}{k_\nu} \right| < 1$: Dynamically active modes $\Omega = \nu k_\nu^2$

Instability



Instability



Instability

Click!

Click!

Conclusions

- New type of Linear Instability: Visco-rotational shear instability
  - Shear rheology + Keplerian differential rotation
  - Unstable modes:
    - Small radial size
    - Vertically uniform
- Analytic solution
- Rheologic correction of epicyclic frequency
- Stability criteria

$$\left(\frac{\partial \ln \eta}{\partial \ln \xi}\right)_P > 2$$

- (Poniatowski & Tevzadze "*Visco-rotational shear instability of Keplerian granular flows*")

Conclusions

1. Pressure rheology:
   - enhance instability: $G_p < 0$
   - reduce instability: $G_p > 0$
2. May lead to:
   a) Nonlinear saturation
   b) Delocalization, *i.e structure formation:*
      - formation of observed patterns in Saturns rings
      - development of planesasimals in protoplanetary discs (?)

- Further analysis needed

Future Work

- 3D compressible
  - Spiral wave instability $\rightarrow$ Spiral density wave instability
  - Comparison with observations
- Numerical Simulations

  PLUTO code.

  Implementation of new RHEO module.
- MSc Project

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