Lecture 4

Finite Volume

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$$\frac{\partial}{\partial t}\int_{\Omega} U \, d\Omega + \int_{\Omega} \vec{\nabla} \cdot \vec{F} \, d\Omega \; = \; 0$$

$$\frac{\partial}{\partial t}\int_{\Omega} U \, d\Omega + \oint_{\partial\Omega} \vec{F} \cdot \vec{n} \, d\Gamma \; = \; 0$$

Finite Volume

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$$\oint_{\partial\Omega} \vec{F} \cdot \vec{n} \, d\Gamma \; \approx \; \sum_{\text{faces}} \vec{F}_k^* \cdot \vec{n}_k$$

$$U_i^{n+1} \; = \; U_i^n - \frac{\Delta t}{V_i} \sum_{\text{faces}} \vec{F}_k^* \cdot \vec{n}_k$$

Finite Volume

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$$\begin{aligned} U_{ijk}^{n+1} \; = \; U_{ijk}^{n} &- \frac{\Delta t}{\Delta x}\left(\vec{F}^*_{i+1/2jk} - \vec{F}^*_{i-1/2jk}\right) \\ &- \frac{\Delta t}{\Delta y}\left(\vec{G}^*_{ij+1/2k} - \vec{G}^*_{ij-1/2k}\right) \\ &- \frac{\Delta t}{\Delta z}\left(\vec{H}^*_{ijk+1/2} - \vec{H}^*_{ijk-1/2}\right) \end{aligned}$$

Finite Volume

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$$\sum_{\text{volumes}} \left(\sum_{\text{faces}} \vec{F}_k^* \cdot \vec{n}_k \right)_i = \sum_{\text{boundary}} \vec{F}_k^* \cdot \vec{n}_k$$

Finite Volume

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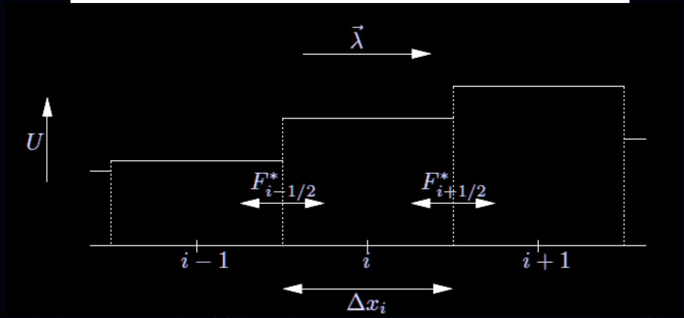
$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x_i}(F^*_{i+1/2} - F^*_{i-1/2})$$



Finite Volume

Central Difference

$$F^*_{i+1/2} = \frac{1}{2}(F(U_i) + F(U_{i+1})) \quad \text{or} \quad F^*_{i+1/2} = F\left(\frac{U_i + U_{i+1}}{2}\right)$$

Lax-Wendroff

$$F^*_{i+1/2} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{\lambda_{i+1/2}\Delta t}{2\Delta x_{i+1/2}}(F(U_{i+1}) - F(U_i))$$

Upwind

$$F^*_{i+1/2} = \begin{cases} F(U_i) & \text{if } \lambda_{i+1/2} \geq 0 \\ F(U_{i+1}) & \text{if } \lambda_{i+1/2} < 0 \end{cases}$$

Finite Volume Method

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$$\frac{\partial}{\partial t}\int_\Omega U \, d\Omega + \int_\Omega \vec{\nabla} \cdot \vec{F} \, d\Omega = 0$$

n, using the Gauss Divergence

$$\frac{\partial}{\partial t}\int_\Omega U \, d\Omega + \oint_{\partial\Omega} \vec{F} \cdot \vec{n} \, d\Gamma = 0$$

$$\frac{\partial}{\partial t}\int_\Omega U \, d\Omega + \int_{X_L}^{X_R} F_x \, dx = \frac{\partial}{\partial t}\int_\Omega U \, d\Omega + (F_R - F_L) = 0$$

Finite Volume

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The flux terms are discretised by

$$\oint_{\partial\Omega} \vec{F} \cdot \vec{n}\, d\Gamma \approx \sum_{\text{faces}} \vec{F}_k^* \cdot \vec{n}_k$$

where $\vec{F}^*$ is known as the **numerical flux.**

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{V_i} \sum_{\text{faces}} \vec{F}_k^* \cdot \vec{n}_k$$

numerical fluxes

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- **Central differences:**

$$F^*_{i+1/2} = \frac{1}{2}(F(U_i) + F(U_{i+1})) \quad \text{or} \quad F^*_{i+1/2} = F\left(\frac{U_i + U_{i+1}}{2}\right)$$

- **Lax-Wendroff:**

$$F^*_{i+1/2} = \frac{1}{2}(F(U_i) + F(U_{i+1})) - \frac{\lambda_{i+1/2}\Delta t}{2\Delta x_{i+1/2}}(F(U_{i+1}) - F(U_i))$$

- **Upwind:**

$$F^*_{i+1/2} = \begin{cases} F(U_i) & \text{if } \lambda_{i+1/2} \geq 0 \\ F(U_{i+1}) & \text{if } \lambda_{i+1/2} < 0 \end{cases}$$

$\lambda = \partial F / \partial U$ is the associated “wave” velocity.

Finite Volume

Finite Difference

- Problems with conservation
- Irregular geometries

Finite Volume

- problems towards the edges

end

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