ლექცია 7

N-Body Problem

Fields: Eulerian description

Bodies: Lagrangian description

One particle:

$$d^2 r / d t^2 = f$$

Interaction of **N** particles

$$d^2 r_i / d t^2 = \Sigma f_i$$

Forces: Gravity, Electrostatic (+/-), …

N-Body Problem

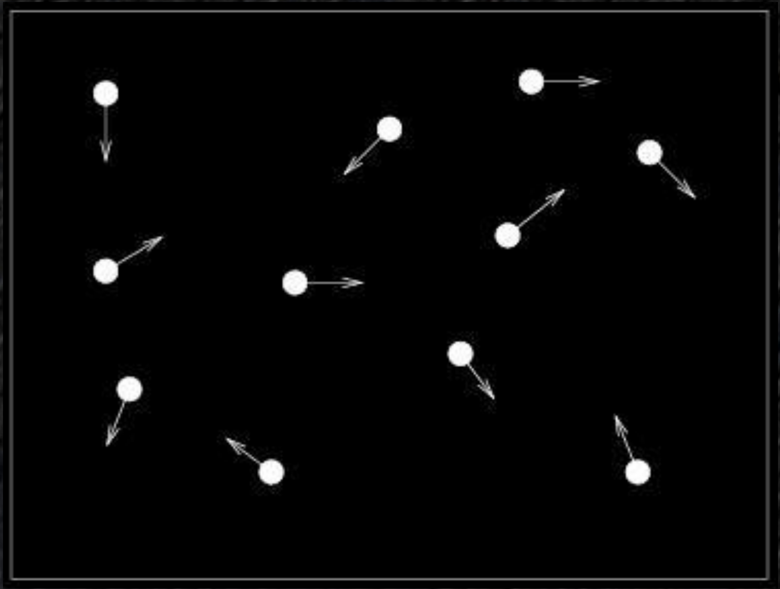
Applications:

- Astronomy: *Formation of Galaxies*
- Cosmology: *Expansion of the Universe*
- Plasma Physics: *Charged Particle Simulations*
- Hydrodynamics: *Dynamics of Point Vortices*
- *Generalized Hamiltonian Systems*

- Biology/Chemistry: *Molecular Dynamics*
- Ecology: *Animal, Plant Distribution*

N-Body Problem

Dynamics of N Particle



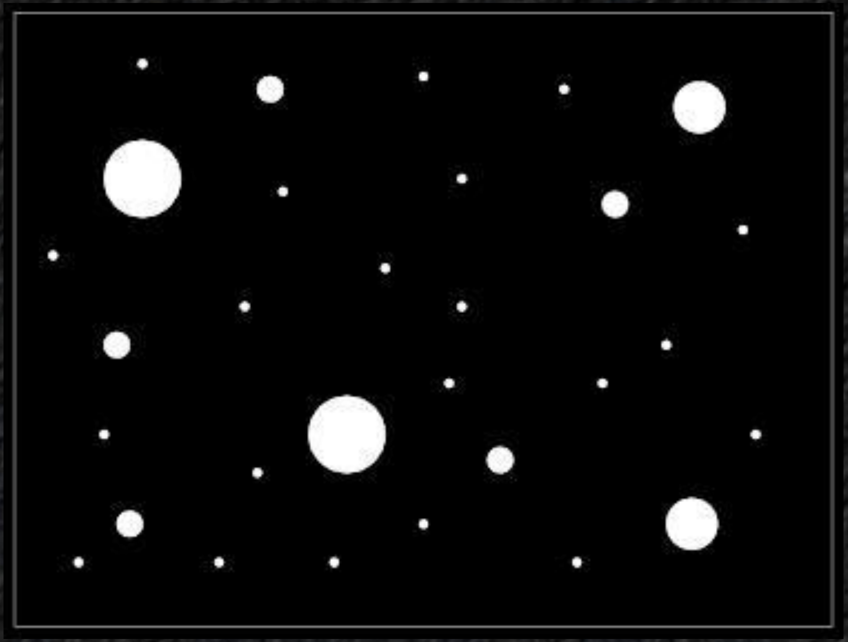
Initial Conditions:

(X_0, V_0)

1. Δt
2. Update $r_{i+1} = r_i + v_i \Delta t$
3. Calculate $f = \Sigma f_i$
4. Update $v_{i+1} = v_i + \Delta t\, f/m$

Particle-Particle **(PP)**

Straightforward Method



1. **Shorter time step for close particles (Δt);**
2. **Collision Rules ($\Delta r = r_{min}$)**

Number of interactions per time step

$$N(N-1) \sim O(N^2)$$

(!!!)

N-Body: Approximations

Methods:

Raw:	PP	
Other:	PM	(1970)
	P3M	(1980)
	PM2	
	TC	(1985)
	NGPM	(1988)
	FMM	(1988)

- Mesh Based Algorithms
- Tree Based Algorithms
- Multipole Based Algorithms

Method: Mesh

<u>Uniform grid</u>

Construct mass density over mesh: $\rho(x)$

Potential over mesh: $\phi(x)$

Poisson Equation

$$\nabla^2 \phi(\vec{x}) = 4\pi G \rho(\vec{x})$$

<u>Force:</u> Particle in potential field $\phi(x)$

Method: Mesh

Accuracy requirement:

1 Particle : (1+) cells

N – Number of particles

m – Mesh pixels

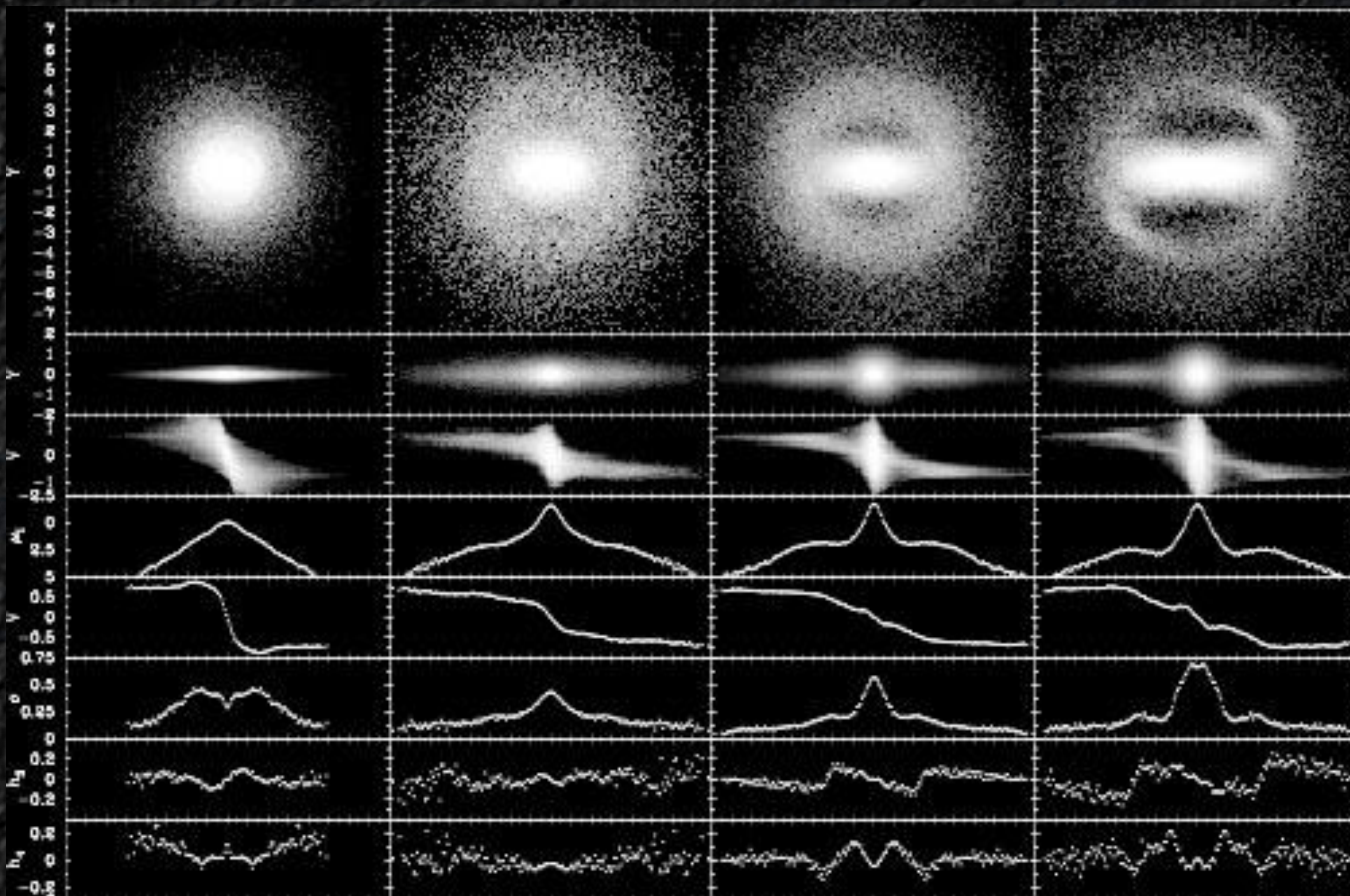
Mesh based method (FFT): *O(m log(m))*

Problem: Nonuniform distribution

$m >> N$: $m \log(m) > N^2$

Method: Mesh

Bar Galaxy Simulations: Particles + Potential

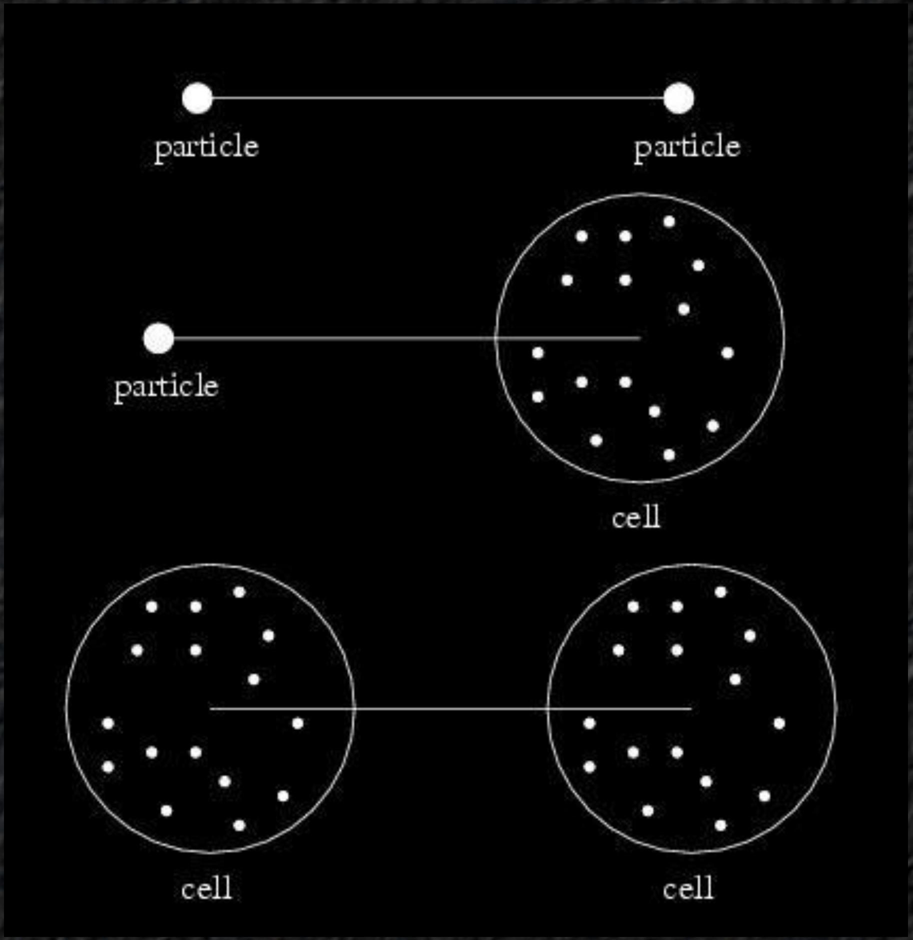


Method: Tree

Introduce **cells**

Particles inside the cell:
cell mass center

Mass center hierarchy:
Tree



Method: Tree

Hierarchy:

divide-and-conquer

Long distance:	Bigger Cell, Mass Center
Short distance:	Smaller Cell, Mass Center
Min distance:	Particle

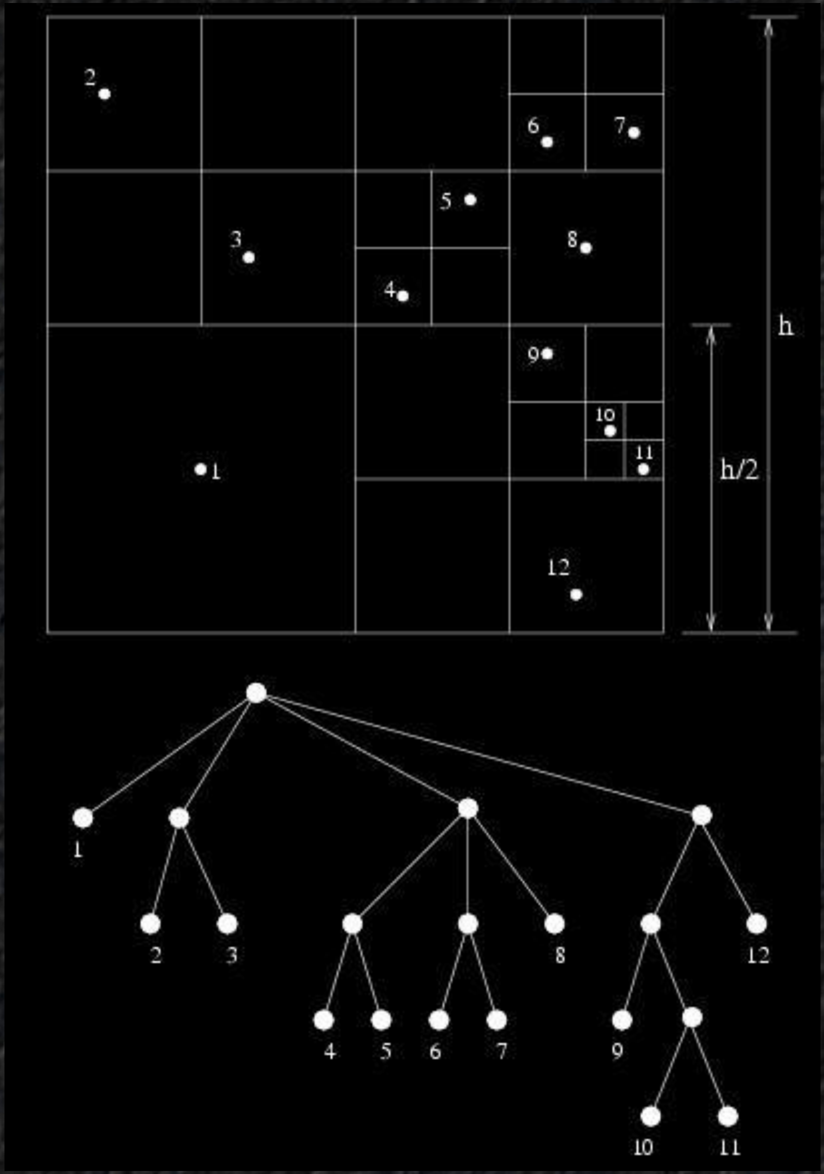
Burnes-Hut algorithm

Method: Burnes-Hut Tree

Quadratee

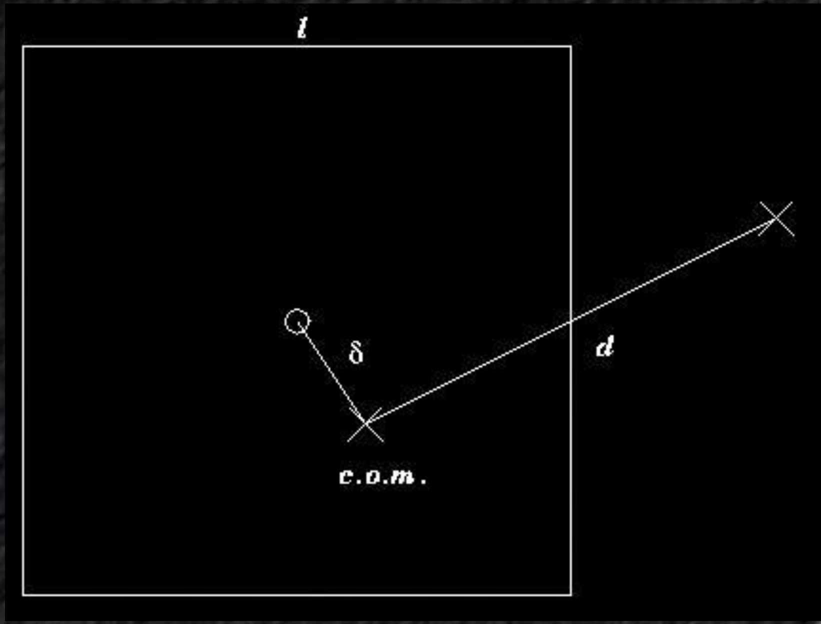
Partition

2D Hierarchy Tree



Method: Burnes-Hut Tree

Cell Opening Criterion:



L – cell size

θ – opening angle parameter

(0 < θ < 1)

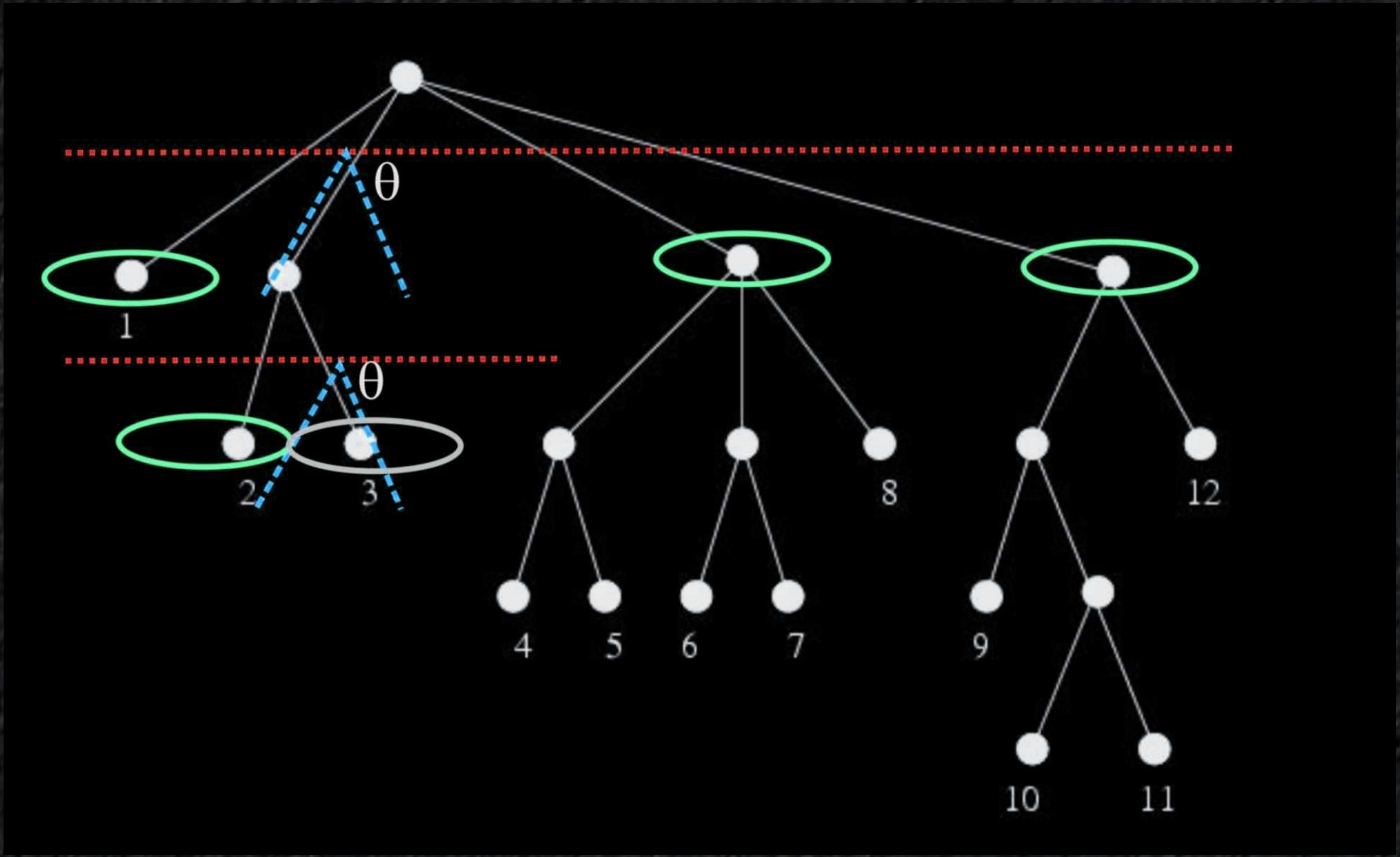
d – distance:

(particle – cell mass center)

d < L/θ

Method: Burnes-Hut Tree

Reconstructing Forces



Method: Burnes-Hut Tree

Cost:

θ **= 0.3**

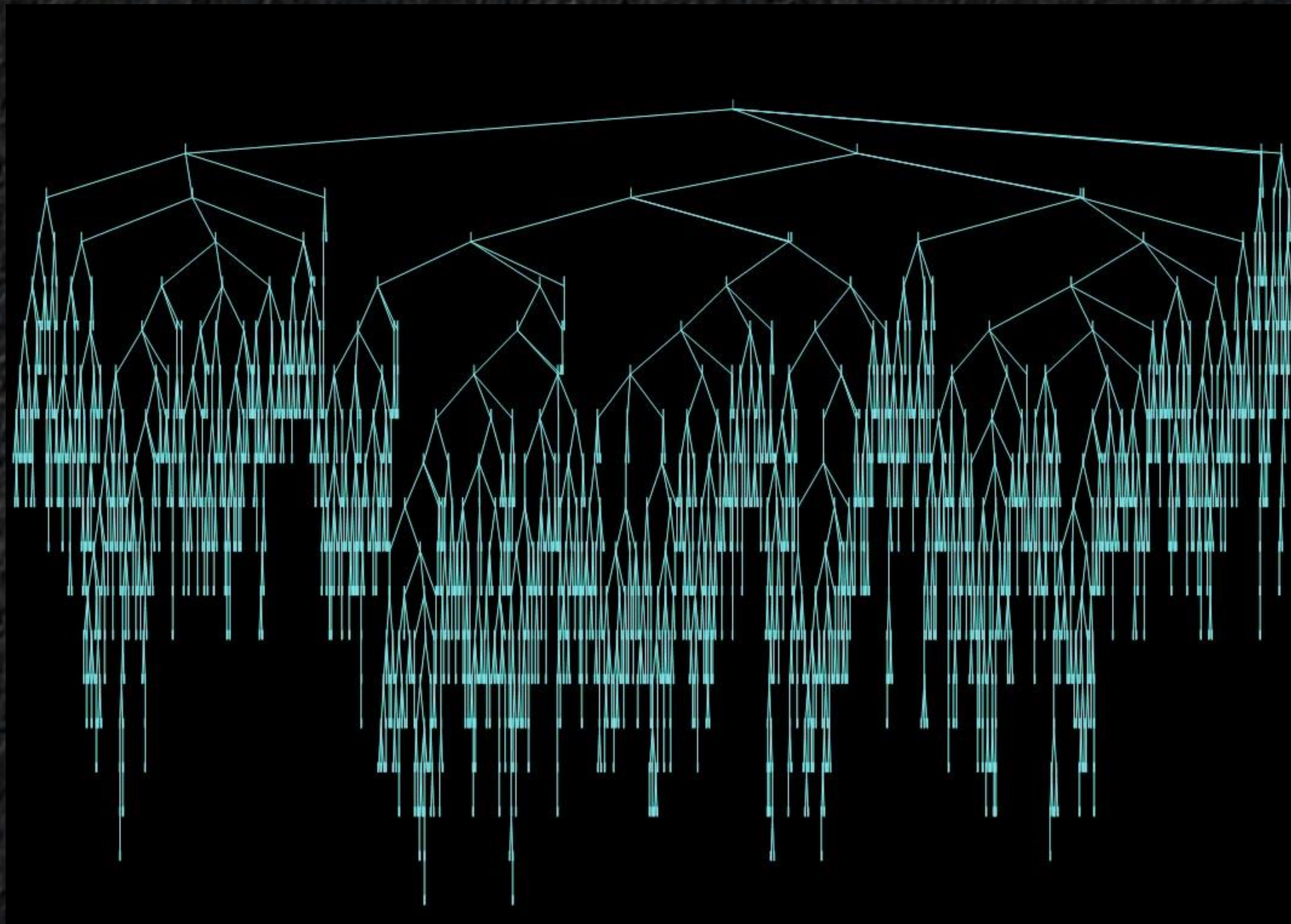
Number of Interactions: $Ni \sim 28\,\pi / 3\,\theta^3\,N \log(N)$

$Ni < N^2 \; : \; N>1000$

Best for higher number of particles

Method: Tree

Example: Barnes-Hut tree



Method: Multipole

Any cluster of particles:

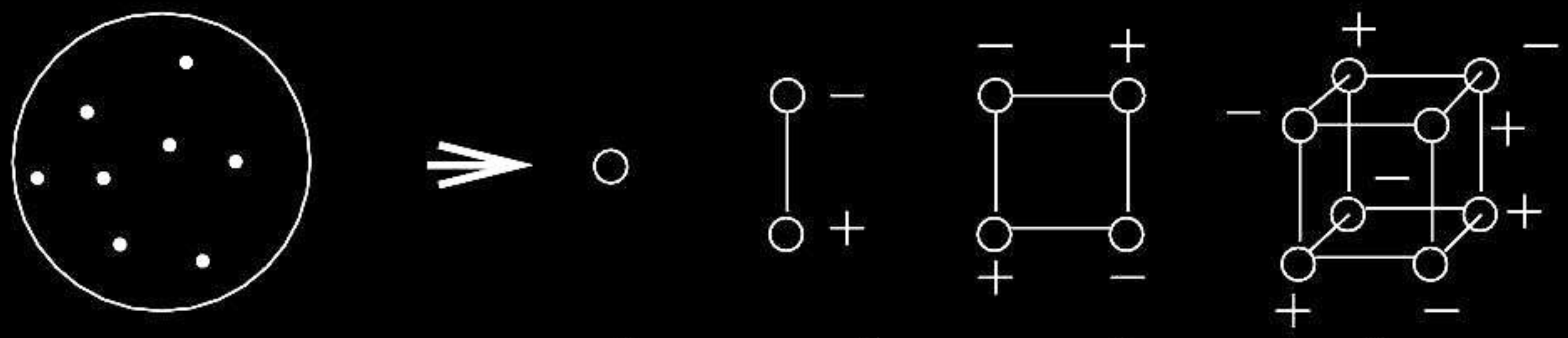
Net force - multipole expansion

Monopole, Dipole, Quadrupole, etc.

- Multipole expansion
- Error bounds on dropping terms

Method: Multipole

Multipole components



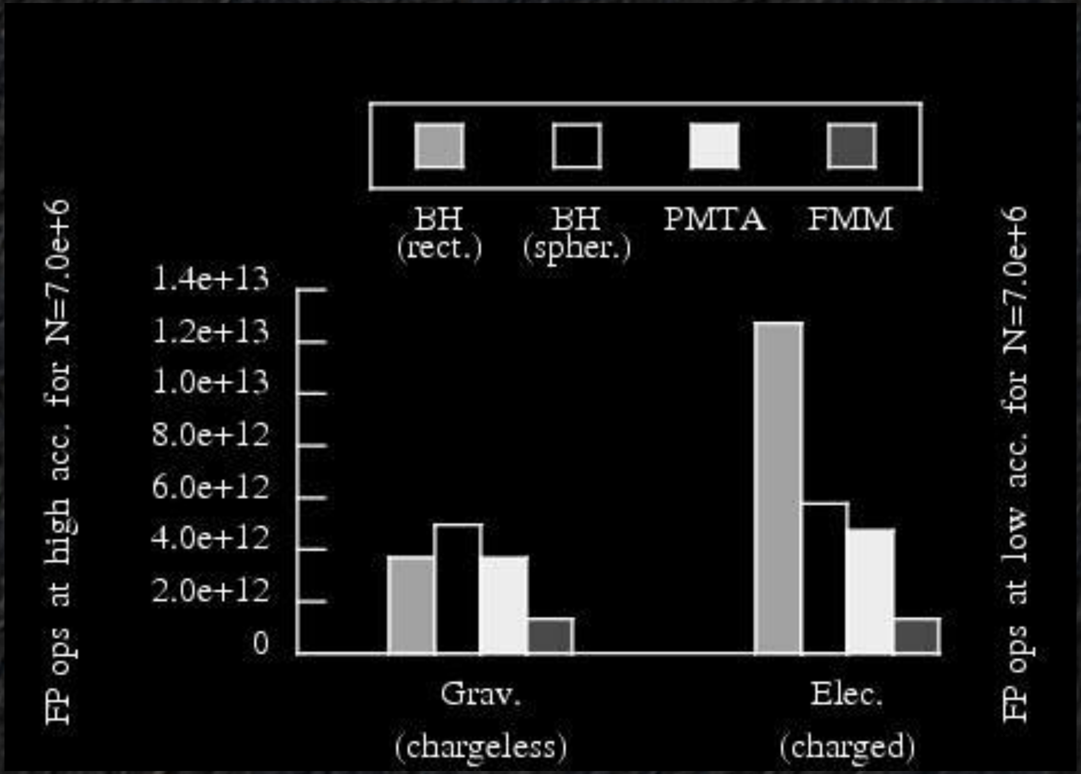
p	1	2	3	4
Terms	1	3	5	7
Potential	$\frac{1}{r}$	$\frac{1}{r^2}$	$\frac{1}{r^3}$	$\frac{1}{r^4}$
Force	$\frac{1}{r^2}$	$\frac{1}{r^3}$	$\frac{1}{r^4}$	$\frac{1}{r^5}$

Method: Multipole

Tree Hierarchy + Multipoles

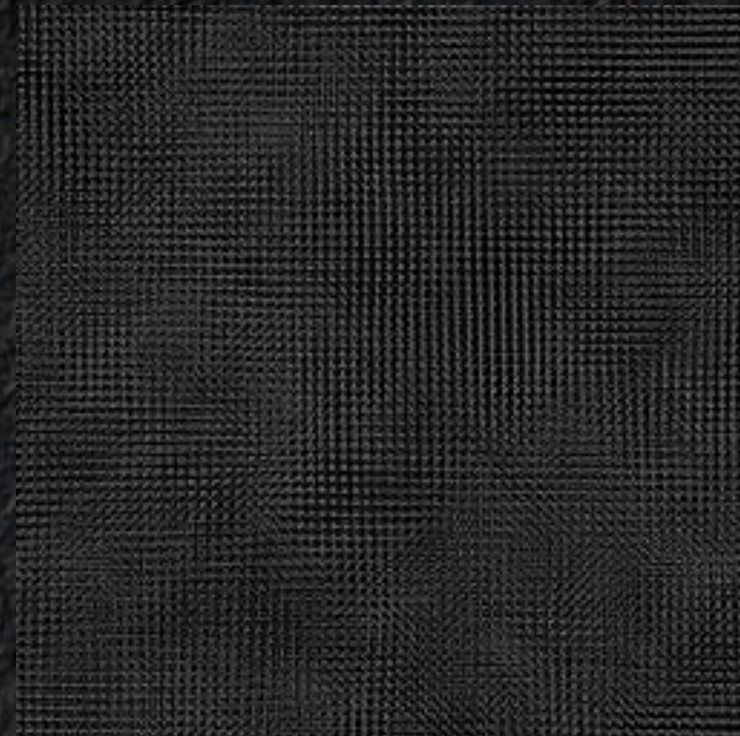
Fast Mutipole Method (FMM)

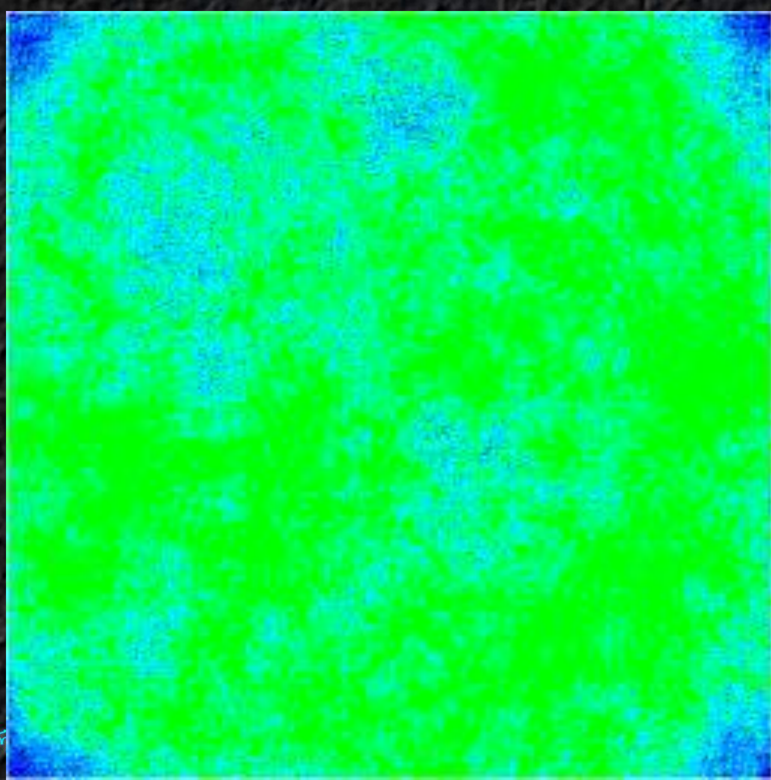
Multipole Expansion: Cartesian, Spherical, …



Galaxy

Formation of the Galaxy Cluster





Cold Dark Matter Cosmology Simulation 17 million particles

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