

რიცხვითი წარმოებულები

$$f'(x) \approx \frac{f(x+h) - f(x)}{h},$$

ტეილორის მწკრივად გაშლა

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(\xi), \quad \xi \in (x, x+h).$$

$$f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h}{2}f''(\xi), \quad \xi \in (x, x+h).$$

Forward difference, backward difference;

$$f'(x) \approx \frac{f(x) - f(x-h)}{h},$$

ცენტრალური წარმოებული:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}.$$

გამოვითვალთ დისკრეტული წარმოდგენის ცდომილება:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(\xi_1),$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(\xi_2).$$

$$\xi_1 \in (x, x+h)$$

$$\xi_2 \in (x-h, x).$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12}[f'''(\xi_1) + f'''(\xi_2)],$$

$$f'''(\xi) = \frac{1}{2}[f'''(\xi_1) + f'''(\xi_2)].$$

$$\xi \in (x-h, x+h)$$

საბოლოოდ:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{6} f'''(\xi),$$

მეორე რიგის წარმოებული:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

$$f(x \pm h) = f(x) \pm hf'(x) + \frac{h^2}{2} f''(x) \pm \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^{(4)}(\xi_{\pm}).$$

$$\frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x) + \frac{h^2}{24} (f^{(4)}(\xi_-) + f^{(4)}(\xi_+)) = f''(x) + \frac{h^2}{12} f^{(4)}(\xi),$$

ცდომილება:

$$-\frac{h^2}{12} f^{(4)}(\xi), \quad \xi \in (x-h, x+h).$$

5 წერტილიანი ფორმულა (5 point stencil)

$$\{x-2h, x-h, x, x+h, x+2h\}.$$

$$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

$$f(x \pm h) = f(x) \pm hf'(x) + \frac{h^2}{2} f''(x) \pm \frac{h^3}{6} f^{(3)}(x) + O_{1\pm}(h^4). \quad (E_{1\pm}).$$

$$f(x+h) - f(x-h) = 2hf'(x) + \frac{h^3}{3} f^{(3)}(x) + O_1(h^4). \quad (E_1).$$

$$f(x \pm 2h) = f(x) \pm 2hf'(x) + 2h^2 f''(x) \pm \frac{4h^3}{3} f^{(3)}(x) + O_{2\pm}(h^4). \quad (E_{2\pm})$$

$$f(x+2h) - f(x-2h) = 4hf'(x) + \frac{8h^3}{3}f^{(3)}(x) + O_2(h^4). \quad (E_2).$$

გამოვაკლოთ: $8 \times (E_1) - (E_2)$

$$8f(x+h) - 8f(x-h) - f(x+2h) + f(x-2h) = 12hf'(x) + O(h^4)$$

მაღალი რიგის წარმოებულები (5 წერტილიანი შაბლონი)

$$f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$$

$$f^{(3)}(x) \approx \frac{f(x+2h) - 2f(x+h) + 2f(x-h) - f(x-2h)}{2h^3}$$

$$f^{(4)}(x) \approx \frac{f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x-2h)}{h^4}$$

ორი ცვლადის ფუნქცია

$$f_x(x,y) \approx \frac{f(x+h,y) - f(x-h,y)}{2h}$$

$$f_y(x,y) \approx \frac{f(x,y+k) - f(x,y-k)}{2k}$$

მეორე წარმოებული:

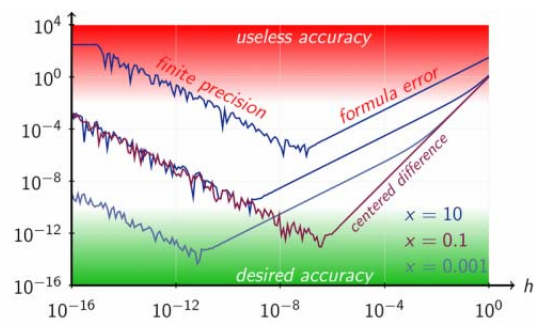
$$f_{xx}(x,y) \approx \frac{f(x+h,y) - 2f(x,y) + f(x-h,y)}{h^2}$$

$$f_{yy}(x,y) \approx \frac{f(x,y+k) - 2f(x,y) + f(x,y-k)}{k^2}$$

შერეული წარმოებული:

$$f_{xy}(x,y) \approx \frac{f(x+h,y+k) - f(x+h,y-k) - f(x-h,y+k) + f(x-h,y-k)}{4hk}.$$

h ბოჯის შერჩევა:



$$h = (\text{epsilon})^{(1/2)} * x$$

epsilon = 2.2e-16; (double precision 64 bit)