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Linear Coupling of Modes in Shear Flows

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Shear Flow Analysis

Linear Mode coupling

Resonant mode conversion

Non-resonant mode conversion

DNS results

Linear mode coupling

Nonlinear developments

Turbulence models

Summary

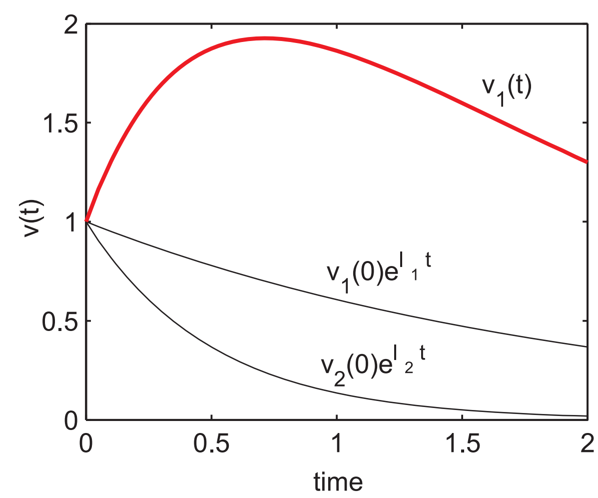
shear flows are non-normal

Non-self adjoint operators;
Eigenfunctions are not ortogonal;

(non-Hermitian system)

Modal analysis fail
(eigenvalue+eigenfunction)

- exponential behavior
- algebraic behavior



Rigorous consideration of non-Hermitian systems

- pseudospectral;

Threfethen et al. 1993

- non-modal analysis;

uniform shear: Kelvin modes,
differential rotation – local frame: Goldreich Lynden-Bell 1965,
kinematically complex shear: Mahajan & Rogava 1999,
nonuniform shear: Volponi & Yoshida 2002

shearing sheet transformation;
spatial inhomogeneity -> temporal inhomgeneity

Spatial Fourier transform;
Dynamics of SFH in time;

$$k=k(t)$$

Linear drift of harmonics; (effect of shearing background)

$$\omega=\omega(t)$$

modes with variable frequencies

modified initial value problem

transient growth (algebraic behavior, flow stability)

Two linear channels of energy exchange:

background flow ←→ perturbations
(WKB, adiabatic, non-adiabatic)

perturbation ←→ perturbations
(different modes; adiabatic,non-adiabatic)

***emphasis on*: Linear Coupling**

Temporal dynamics of SFH

Linear modes are coupled

Velocity shear originates coupling terms in linear equations

two types of coupling:

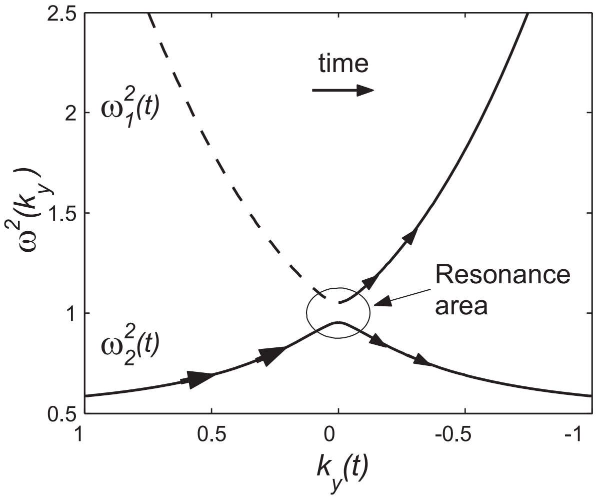
- resonant

(wave-wave)

- non-resonant

(vortex-wave, wave-wave, spectrally unstable modes)

Resonant wave interactions



linear drift of SFH + mode coupling

Linear Mode coupling: Resonant mode conversion

Mathematical formalism:

$$\frac{\mathrm{d}^2\Phi_1(t)}{\mathrm{d}t^2} + \omega_1^2(t)\Phi_1(t) = -\mathbf{\Lambda}\Phi_2(t),$$

$$\frac{\mathrm{d}^2\Phi_2(t)}{\mathrm{d}t^2} + \omega_2^2(t)\Phi_2(t) = \mathbf{\Lambda}\Phi_1(t),$$

Resonance conditions:

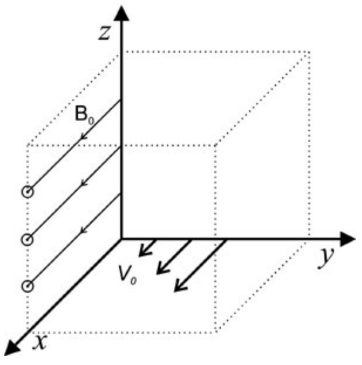
1. the system should have a the degeneracy region, where

$$|\omega_1^2(t) - \omega_2^2(t)| < \left|\frac{\mathbf{\Lambda}\Phi_i(t)}{\Phi_i(t)}\right|,$$

2. the degeneracy region should be crossed slowly:

$$\left|\frac{\mathrm{d}\omega_i(t)}{\mathrm{d}t}\right| \ll \left|\frac{\mathbf{\Lambda}\Phi_i(t)}{\Phi_i(t)}\right|,$$

Linear Mode coupling: Resonant mode conversion



Horizontal shear flow in the uniform magnetic field along the streamlines:

$\mathbf{V_0}=(Ay,0,0)$, $\mathbf{B_0}=(B_0,0,0)$

Unbounded 3D ideal compressible MHD shear flow

☑ Fast magnetosonic and Alfven waves

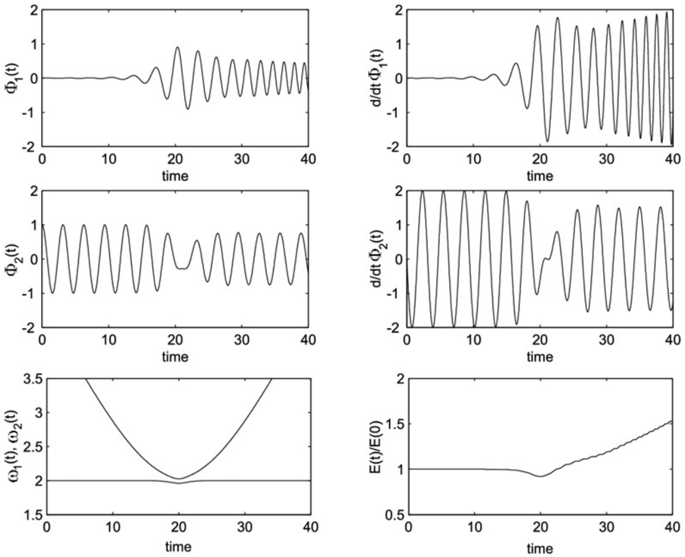
$\omega_f^2 \approx \omega_A^2$ $\quad(\beta < 1,\ k_z / k_x << 1)$

☑ Alfven and slow magnetosonic waves

$\omega_A^2 \approx \omega_s^2$ $\quad(\beta > 1)$

☑ Fast, slow magnetosonic and Alfven waves

$\omega_f^2 \approx \omega_A^2 \approx \omega_s^2$ $\quad(\beta = 1,\ k_z / k_x << 1)$



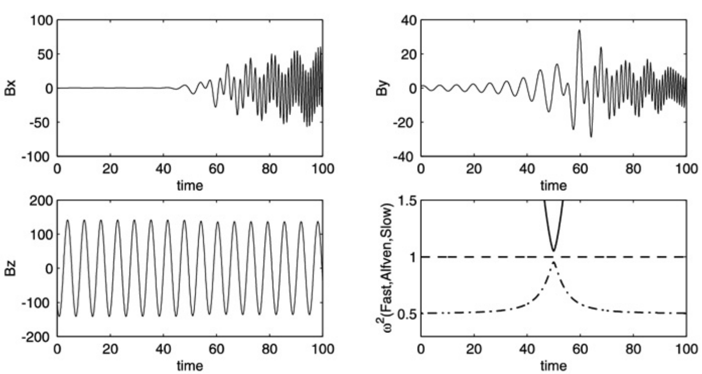
Transformation of the Alfven into the fast magnetosonic wave

$k_y(0)/k_x = 2$,

$k_z/k_x = 0.25$,

$A/(V_A k_x) = 0.025$

Triple Resonance



The case of double transformation: Alfvenic perturbations generates fast and slow magnetosonic waves simultaneously.

$\beta = 1$, $k_y(0)/k_x = 5$, $k_z/k_x = 0.05$,

$A/(C_s k_x) = 0.1$

Linear Mode coupling: Resonant mode conversion

- More complex magnetic configurations;
- Stratification;
- Rotation;
- analytic form of the transformation coefficients; (asymptotic cases)

Direct resonance:

Energy exchange between the linear modes

Resonance conditions:

increase of the shear parameter may decrease of the transformation rate

Coupling formalism

2D unbounded compressible parallel shear flow: **V** = *(Ay,0)*

zero shear limit: (Vortex + Wave)

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} v_x^{(\mathrm{w})}(t) + c_s^2 k^2 v_x^{(\mathrm{w})}(t) = 0$$

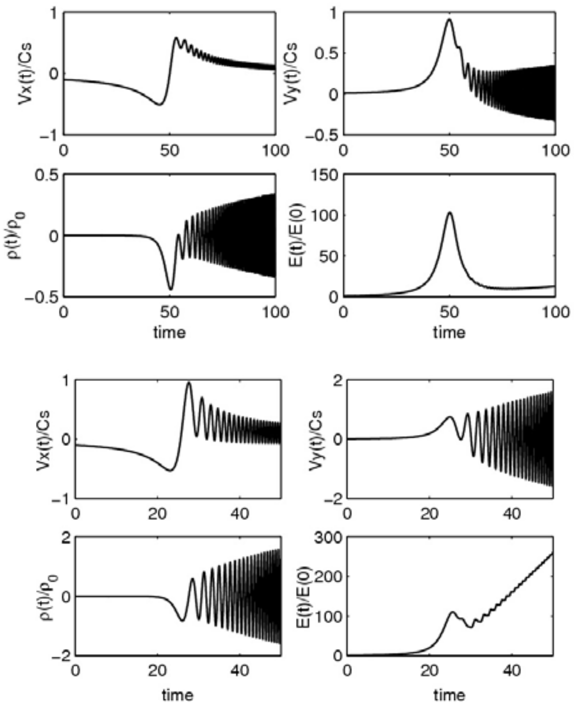
$$k^2 v_x^{(\mathrm{v})} = k_y I.$$

velocity shear induced coupling: (Vortex + Wave)

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} v_x(t) + c_s^2 k^2(t) v_x(t) = c_s^2 k_y(t) I,$$

vortex is able to excite wave: non-resonant interaction

Linear Mode coupling: non-resonant mode conversion



Evolution of vortex SFH in compressible shear flows:

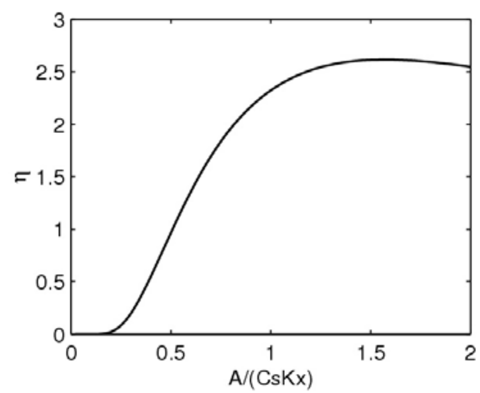
Upper panels:

$A / c_s k_x = 0.2$

Lower panels:

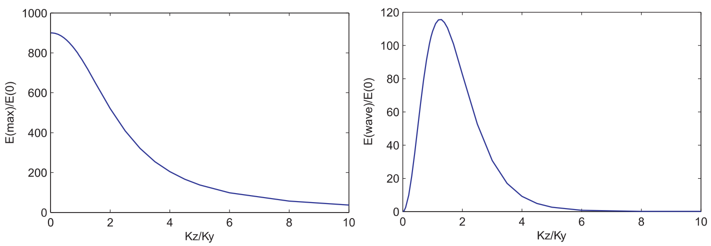
$A / c_s k_x = 0.4$

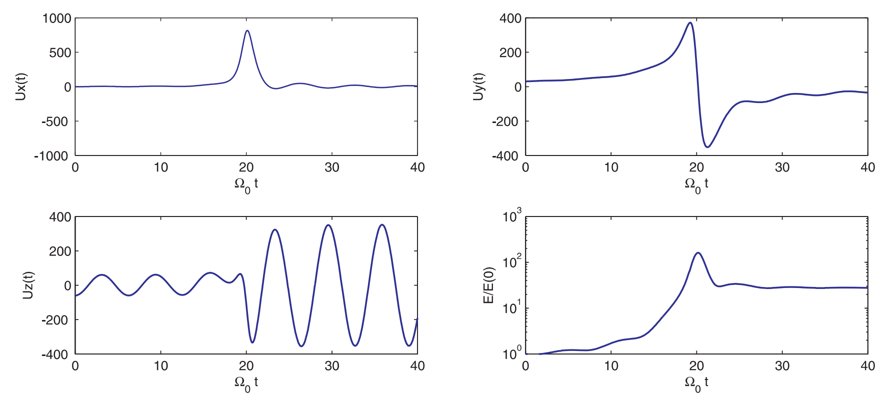
Generated wave amplitude



HD Keplerian disks

Interplay of the transient growth and mode conversion





HD turbulence transition: bypass model

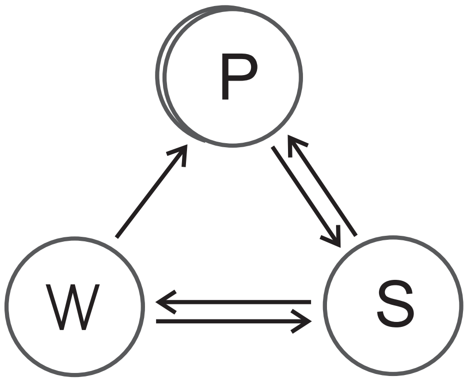
Linear Mode coupling: non-resonant mode conversion

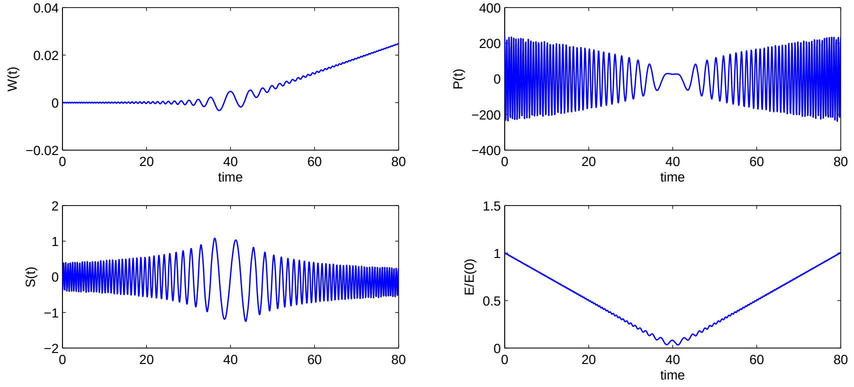
complex systems

vortex generation: baroclinic production;
Differentially rotating disk: Entropy gradient S=S(r)

Multi mode conversion

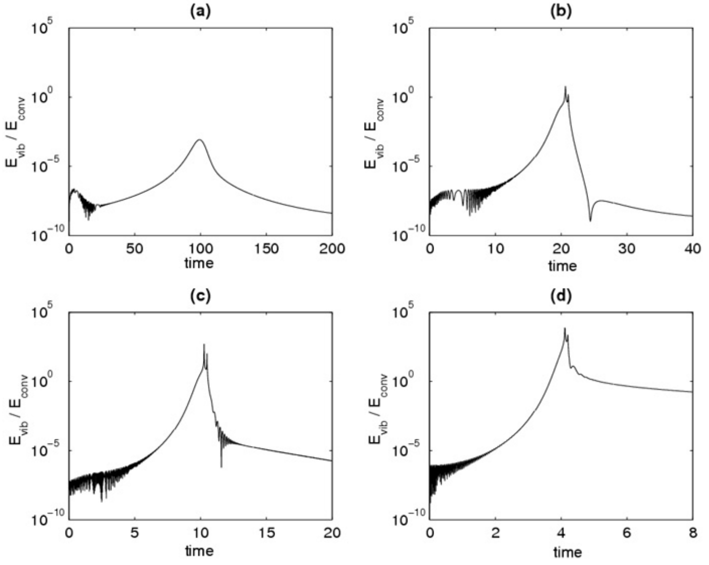
Coupling scheme:





A. G. Tevzadze, ***Linear couplin***

Linear interaction of spectrally unstable modes and waves: wave excitation



Buoyancy -> Waves

The ratio of the vibrational to the thermal energy of the SFH of perturbations (E_{vib}/E_{con}) vs time is shown at different velocity shear rates. Here $K_x = K_z = 10$, $K_y(0) = 200$ and $\sigma = -0.055$ ($\gamma = 0.95$). $R = 0.2, 1, 2, 5$ on the a, b, c and d graphs respectively. The time interval is chosen to show symmetric values of $K_y(\tau)$: $|K_y(0)| = |K_y(2\tau^*)|$ and $K_y(\tau^*) = 0$, where $\tau^* = 100, 20, 10, 4$ on the a, b, c and d graphs, respectively.

Excitation asymmetry: Vortex -> wave

PV conservation prevents generation of the vortices

Excitation rate growth with shear parameter

Wave excitation is quite abrupt

waves are excited when $k_y(t) = 0$

Excited waves are fed by the mean shear flow energy

Generated waves can have more energy then the source vortex: vortical perturbations only trigger the wave excitation while the energy is supported by the mean shear flow

Generated waves are spatially correlated with sources

DNS results

- Linear dynamics of vortices in plane shear flows; mode conversion;

- Dynamics of vortices in Keplerian disks:

global HD simulations
Riemann, Godunov DNS

Linear amplitudes;
Nonlinear consequences;

Transition to turbulence - bypass model

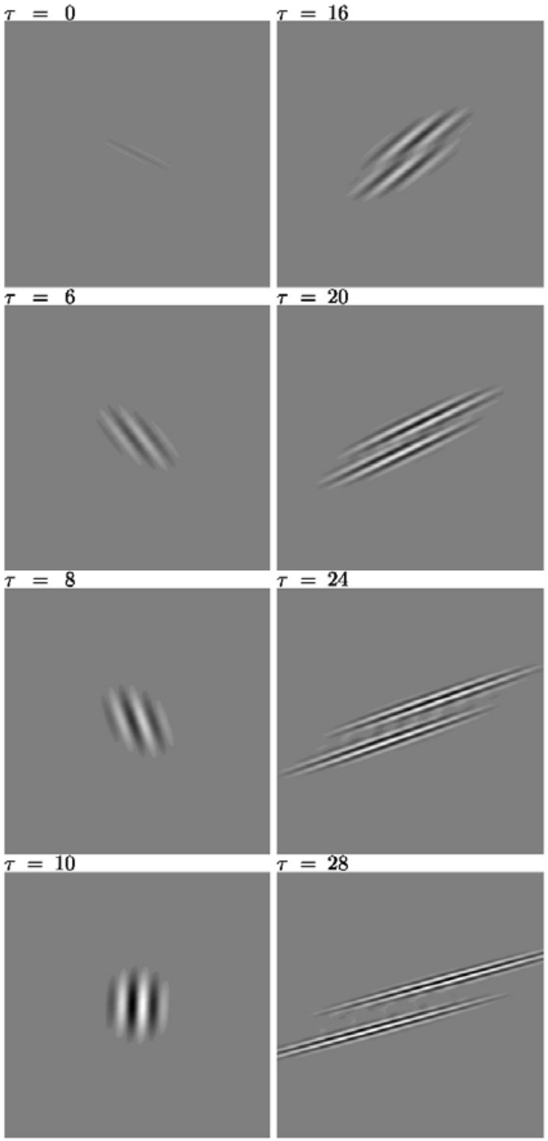
HD nonlinearitiesl pseudospectral code;
transverse cascade

DNS results

Localized vortex packet with linear geometry

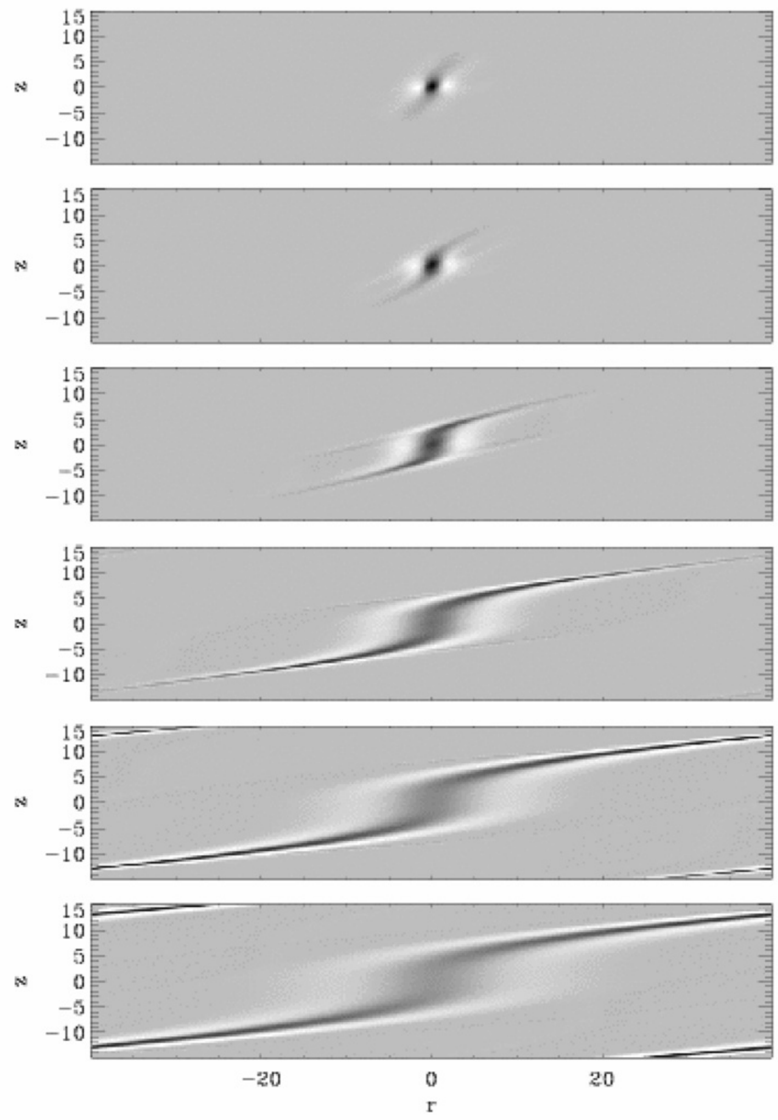
Enhencement of the vortex packet amplitude is followed by the wave excitation

Excited waves propagate in the opposite directions



DNS results

Ring-type vortex

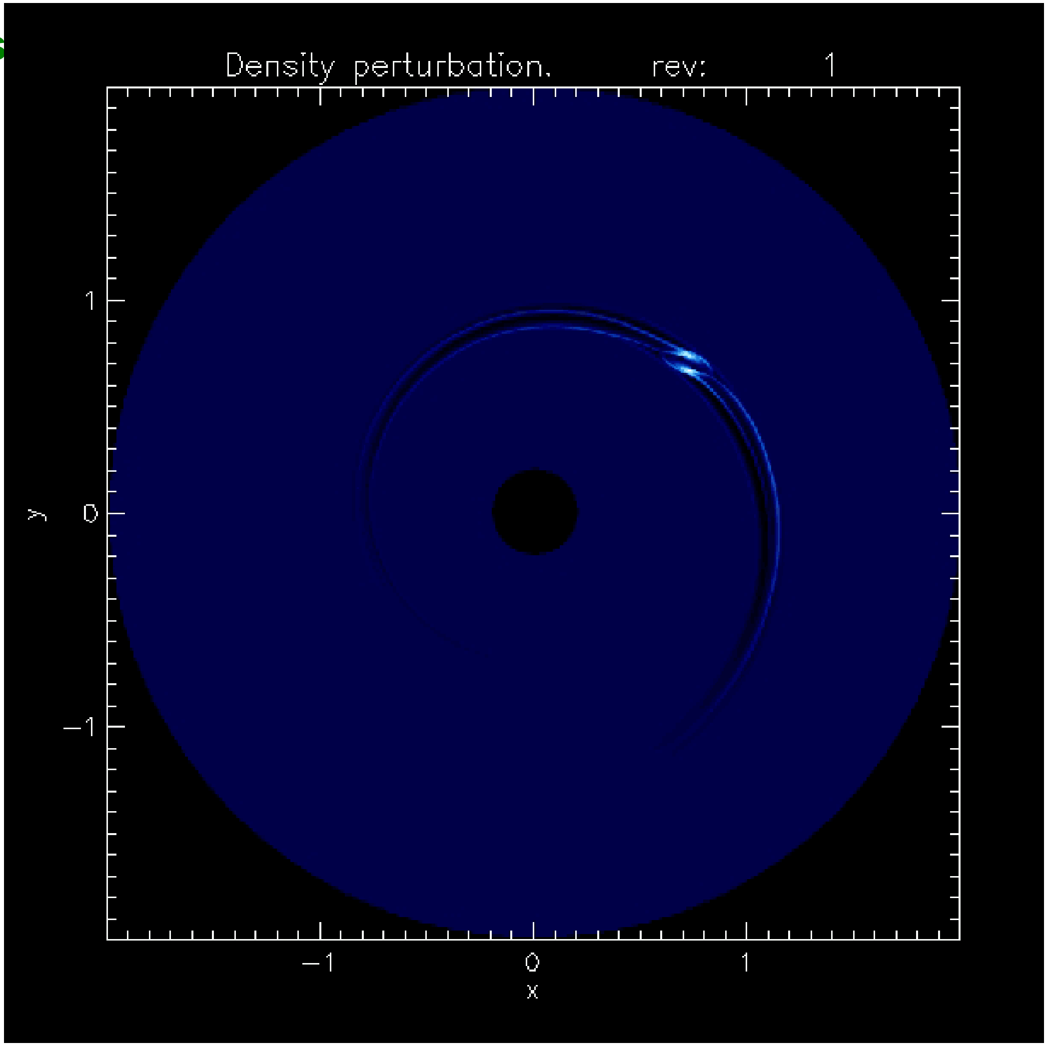


DNS results

Keperian disk flow:

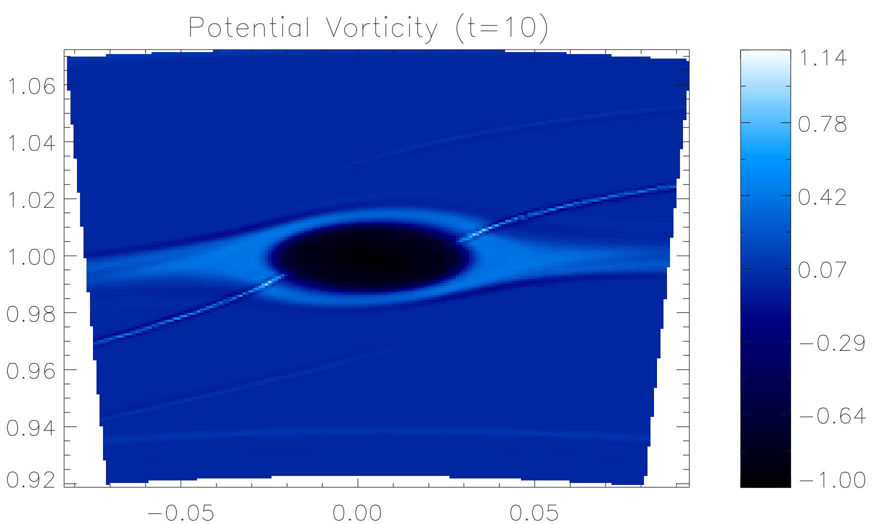
Nonlinear self-sustained vortices
mode conversion

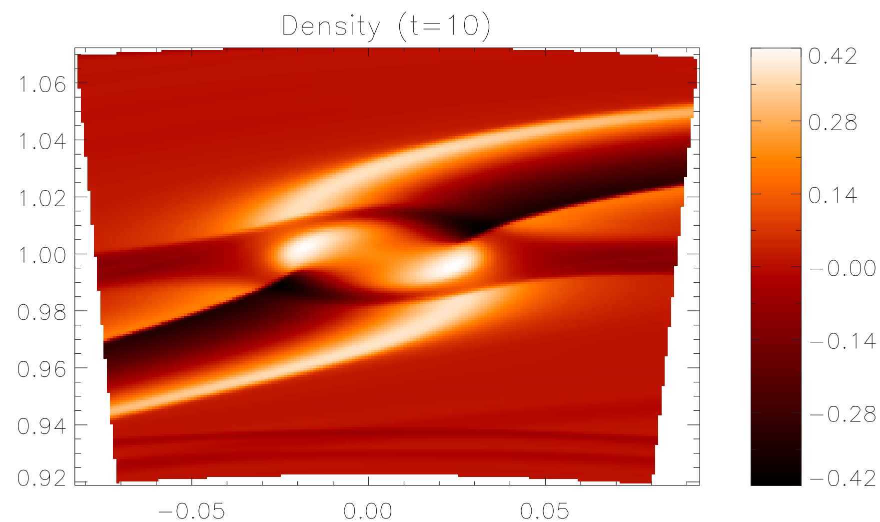
development of shock waves



DNS results

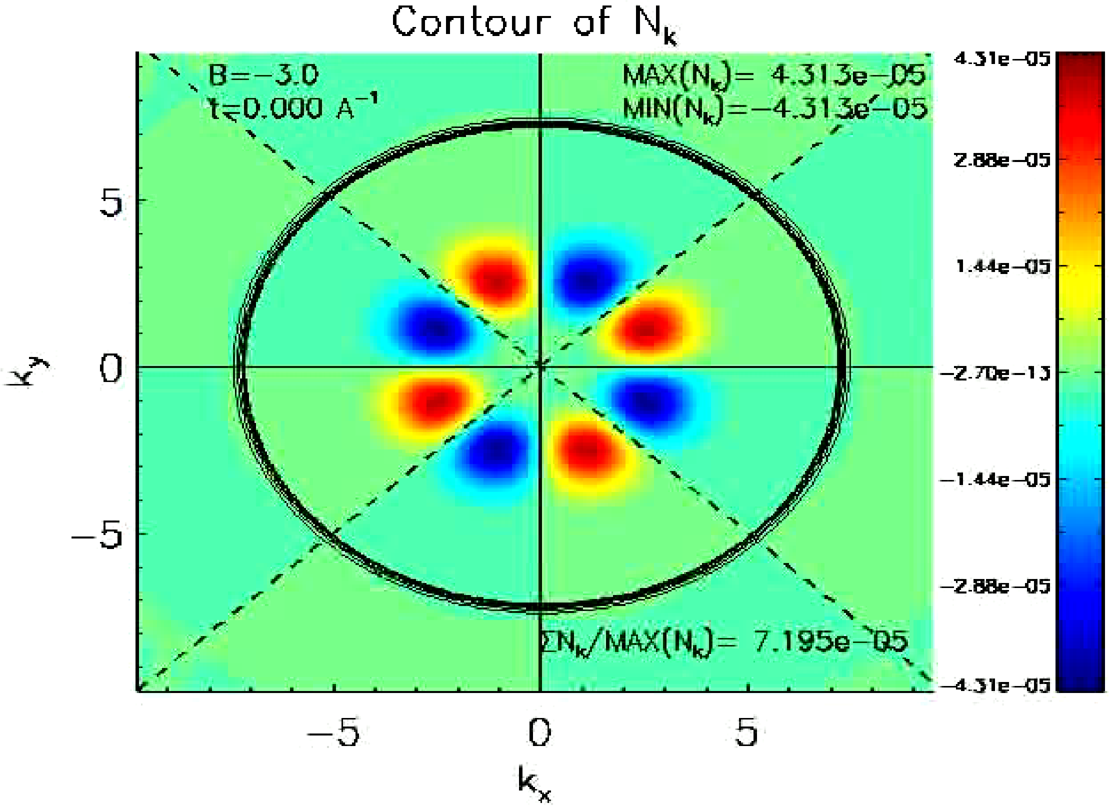
Keperian disk flow:





DNS results

Nonlinear interactions in shear flows: transverse cascade



DNS results

- Plane shear flows

vortices with different configuration generate waves linearly

- Keplerian Disk flows

linear wave excitation
nonlinear excitation
Planet formation models
Shock development

- Nonlinear interaction of modes in shear flows

transverse cascade – bypass model
L-H transition in tokamaks

Summary

multiple brunches in linear spectrum – shear flow couples all of them
(some restrictions due to the nonlinear invariants, e.g. conservation of PV)

modes can fall in resonanse (subject to resonance conditions)

modes interact nonadiabatically at higher shear rates
(trigger excitation, energy comes from background)

new energy channels between intrinsically different modes
(vortices, waves, unstable branches)

mode coupling is efficient even at nonlinear amplitudes

number of applications can play a central role and define the flow structure/stability itself
(vortex stability, HD turbulence, increased energy supply)