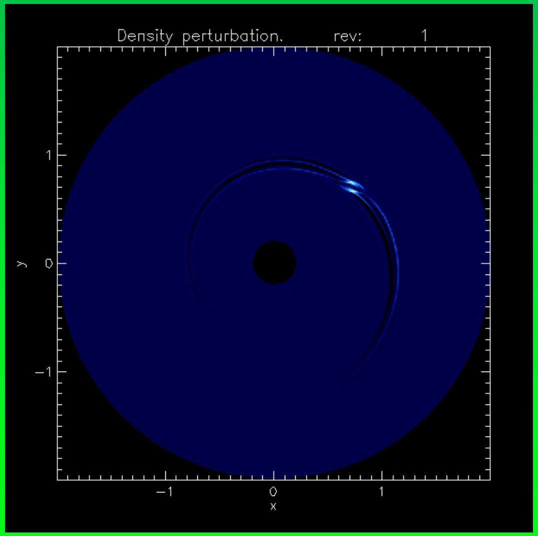
Nonlinear Dynamics of Vortices in 2D Keplerian Disks:

High Resolution Numerical Simulations



- **Vortices in Protoplanetary Disks**
- **Problem Parameters**
- **Numerical Setup**
- **Nonlinear Adjustment**

  threshold, stability, adjustment time-scales

  resolution study, structure/spatial-scales,
  Pot. vorticity and density adjustments

- **Generation of Waves – Nonlin. Development**
- **Summary**
- **Open Issues**

Vortices in Protoplanetary Disks

Protoplanetary Disk: pressure supported { gas + dust + particles }

Planet formation:

- Early stages – need for planetesimals
- Keplerian differential rotation – strong shear flow

Long-lived vortices can promote the formation of planetesimals
(von Weizacker 1944, Adams & Watkins 1995)

Importance of the vorticity: Nonlinear dynamics

Vortices in Protoplanetary Disks

Previous contributions:

Bracco et al. 1998,1999	(incompress. Dust trapping)
Godon & Livio 1999a,b,c	(2D compress. Global, 128x128)
Chavanis 2000	(dust capturing)
Davis 2000,2002	(2D compress. Global,125x300)
Johansen et al. 2004	(dust in anticylones, local)
Barranco & Marcus 2005	(3D spectral, local)
Bodo et al. 2005	(2D global. Linear waves)

- <u>High resolution</u> <u>global</u> <u>compressible</u> <u>2D</u> Numerical simulations of the <u>small scale</u> nonlinear vortex structures

Vortices in Protoplanetary Disks

☞ **Stability of anticyclonic vortices**

Detailed Quantitative Description

☞ **Development Time**

☞ **Developed Vortex Size**

☞ **Developed Vortex Structure**

☞ **Wave Generation Aspects**

☞ **Numerical Requirements**

Problem Parameters: *time-scales*

☞ Linear Perturbations:

$T(SD) \sim 1/\omega_{SD}$

$T(shear) \sim 4/3\Omega_0^{-1}$

$T(R)^* \sim 1/\omega_R$

$$\omega_R^2 = \frac{3}{4}\frac{\Omega_0}{r}\frac{k_y}{k_x^2 + k_y^2},$$

$$\omega_{DS}^2 = \Omega_0^2 + c_s^2(k_x^2 + k_y^2),$$

Vortex dynamics: $T(R)^* >> T(shear) > T(SD)$

☞ Nonlinear Perturbations:

$T(nonlin) \sim$ (nonlinear adjustment / num. simulations)

Vortex dynamics $T(R)^{**} >> T(nonlin.) > T(shear,SD)$

$T(R)^{**} > T(R)^*$

Problem Parameters: *spatial-scales*

H – disk thickness

r – disk radius at vortex

L – vortex length-scale

L_R – Rossby length-scale

Thin disk model:	$H/r \ll 1$
2D Perturbations:	$L > H$
Vortex in disk:	$L < L_R$

$L > H$: $\quad L > C_S/\Omega_0$

$L < L_R$: $\quad \{T_{non}/2\pi\}\,(\Omega_0\, L/r)^{1/2} \ll 1$

$L_R > H$: $\quad C_S < r\,/\,(T_{non}/2\pi)^2 = r\,/\,N^2$;

Numerical Setup

PLUTO

Polar grid – radially stretched

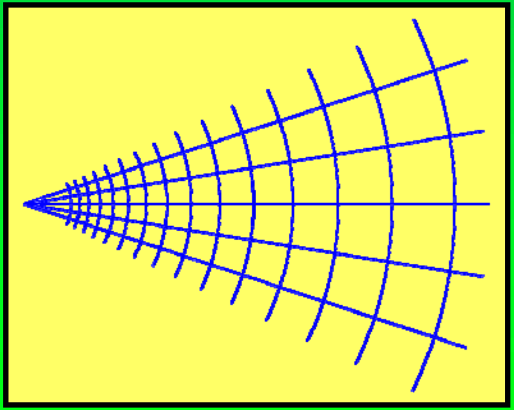
Angular momentum conservation form;

FARGO

Outflow boundary conditions radially

Resolution: $(N_R \times N_\Phi)$

$N_R = f(N_\Phi, R_{in}\ R_{out})$



Numerical Setup

	endian	Polar Rho jpg global/local	rho jpg 2avi	Total # of revolut.	Vort.	Dom.	Notes
E:\disc\vs10-1							
eps01a01R1500x550	ok	+/	0	12		2π	
eps02a01R1500x550	ok	+/	0	12		2π	
eps03a01R1500x550	ok	+/	0	12		2π	
eps05a01R1500x550	ok	+/	0	23		2π	
eps08a01R1500x550	ok	+/	0	12		2π	
E:\disc\vs10-2							
eps01a01r1500x550							
eps01a01r2000x733	swap			10		2π	
eps01a01r4000x1466	swap			7?		2π	
eps1a01r4000x1466	swap			-			
eps02a01r2000x733	ok	+/	0	20		2π	
eps03a01r1000x366	ok	+/	0	20		2π	
eps03a01r2000x733	swap	+/	0	19		2π	
eps03a01r4000x1466	swap	+/		19		2π	
eps05a01r4000x1466	swap	+/		15		2π	
eps022a01r4000x1466	ok			20/1		$\pi/2$	
eps025a01r4000x1466	ok	+/	0	20/1		$\pi/2$	
eps028a01r4000x1466	ok	+/	0	20/1		$\pi/2$	
eps05a001r2000x733	ok	+/		20/1		2π	Resol: ○◎
eps05a005r2000x733	ok	+/		20/1		2π	
eps05a005r2000x733_s	ok	+/		3/8		2π	
eps05a01r2000x733	ok	+/		80/1		2π	
eps05a02r2000x733	ok	+/		20/1		2π	
eps05a001r4000x1466	ok	/+		17/1		2π	Resol: ◎○◎
eps05a005r8000x1559	ok	/+		10/1		2π	
eps05a01r4000x1466				50/1		2π	
eps01a01r4000x1466				20/1		2π	

Initial Conditions

$$v'_x(x,y,0) = \epsilon_r \frac{y-y_0}{b} \exp\left[-\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2)}{b^2}\right]$$

$$v'_y(x,y,0) = -\epsilon_r \frac{x-x_0}{a} \exp\left[-\frac{(x-x_0)^2}{a^2} - \frac{(y-y_0)^2)}{b^2}\right]$$

$$\rho'(x,y,0) = 0$$

Circular vortex: *a*=*b*

Elliptic vortex: *a*/*b* = *q* (aspect ratio)

Nonlinear Adjustment

Transition from the initial unbalanced to the final nonlinear self-sustained configuration

- ✓ Initial Imbalance (amount of nonlinearity, ε)
- ✓ Spatial Scale (initial vortex size, a)
- ✓ Adjustment time (Resolution, Cs)

long-lived self-sustained anticyclonic vortex

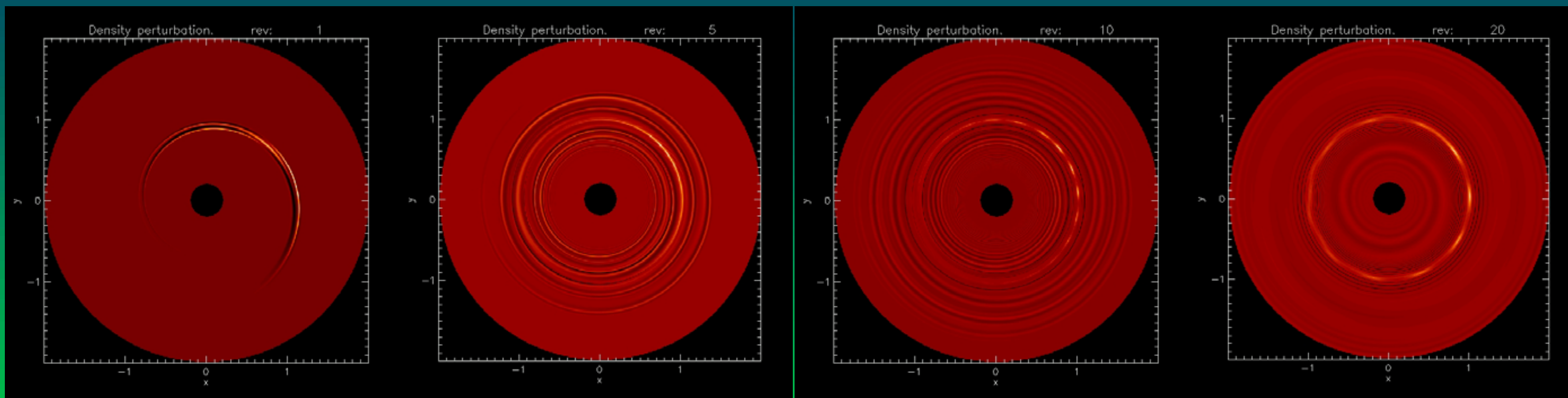
nonlinear balance configuration

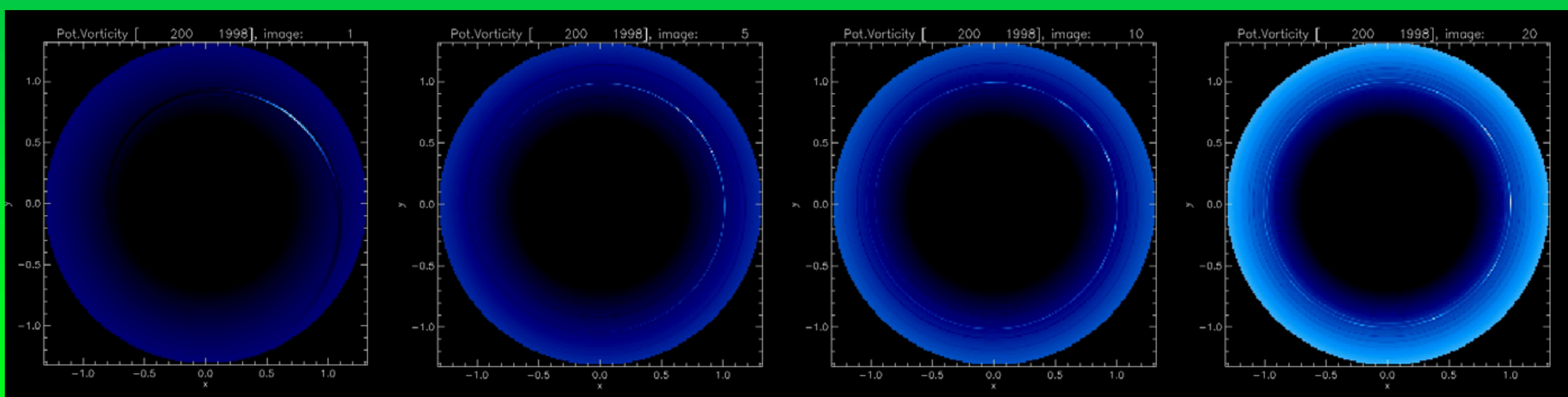
Threshold Value

Amplitude of initial vortex - two nonlinear thresholds:

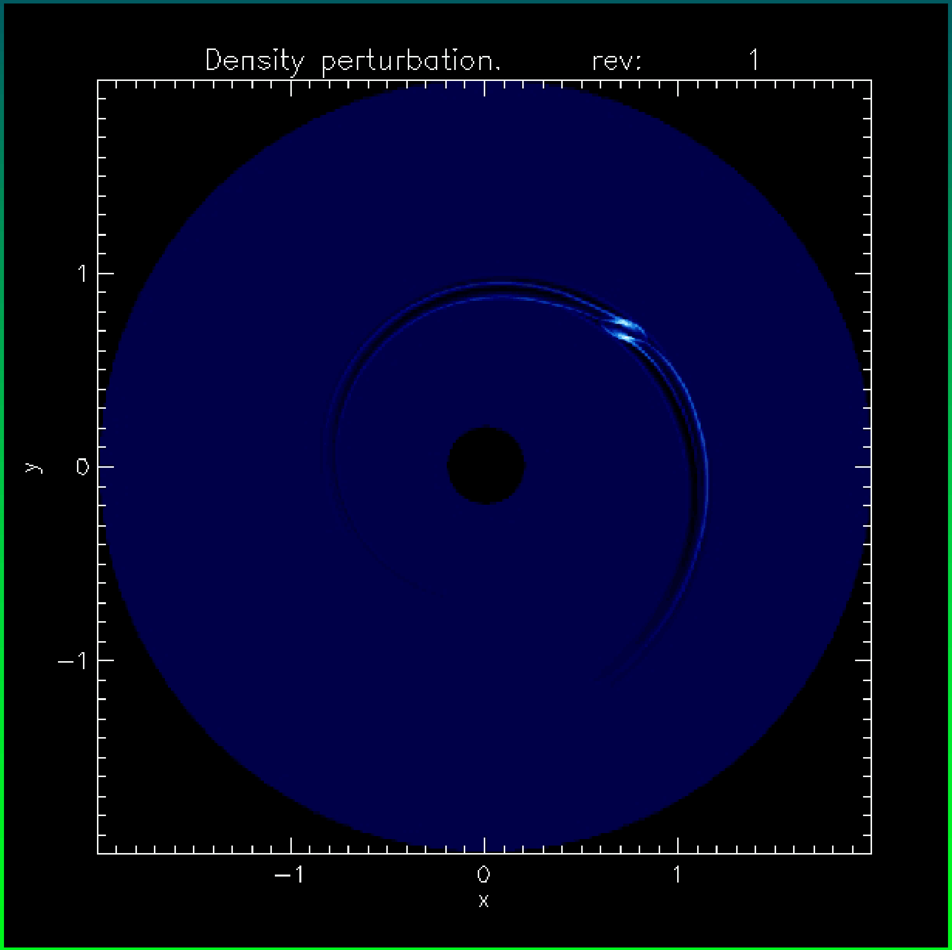
1. **Linear case:** $\varepsilon < 0.1$
2. **Weakly nonlinear case, not sufficient for direct adjustment** $0.1 < \varepsilon < 0.25$
3. **Strongly nonlinear case, direct adjustment to single vortex** $0.25 < \varepsilon$

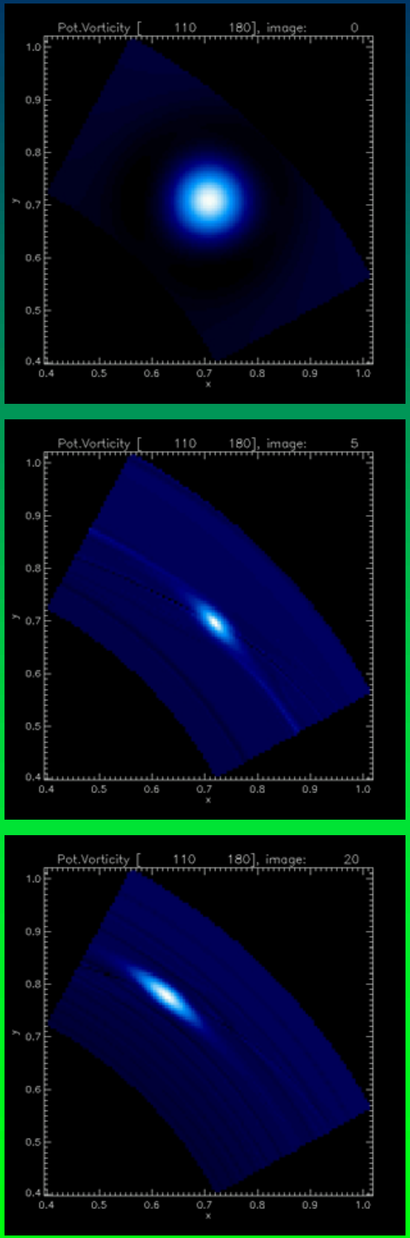
eps = 0.2 a = 0.1 res: (2000x733)





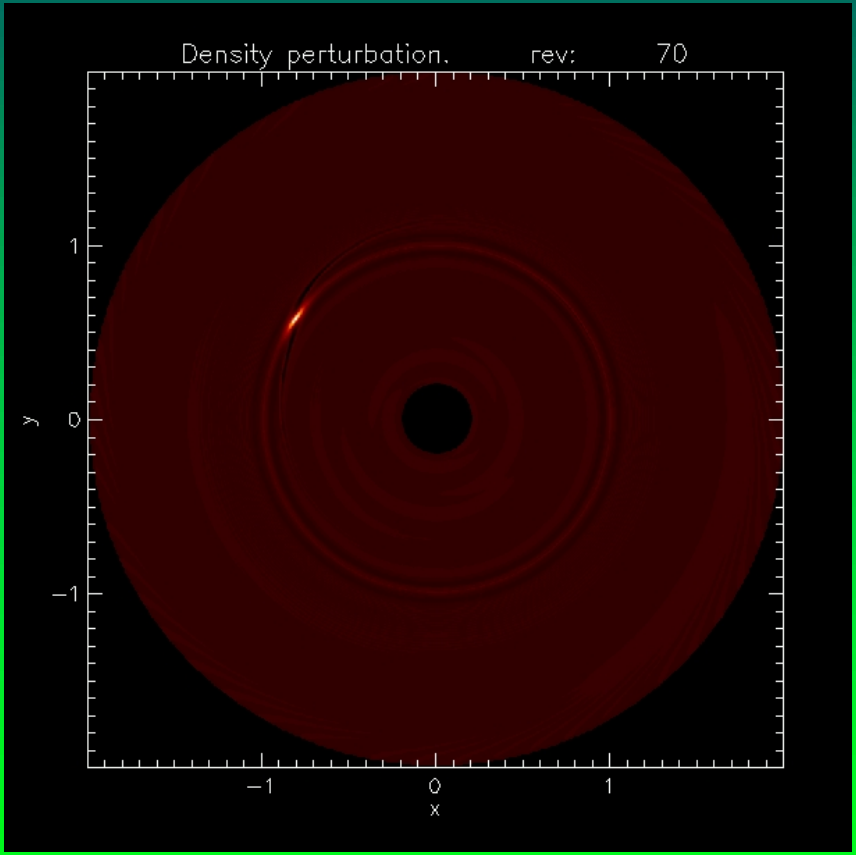
eps = 0.5 a = 0.1 res: (2000x733)

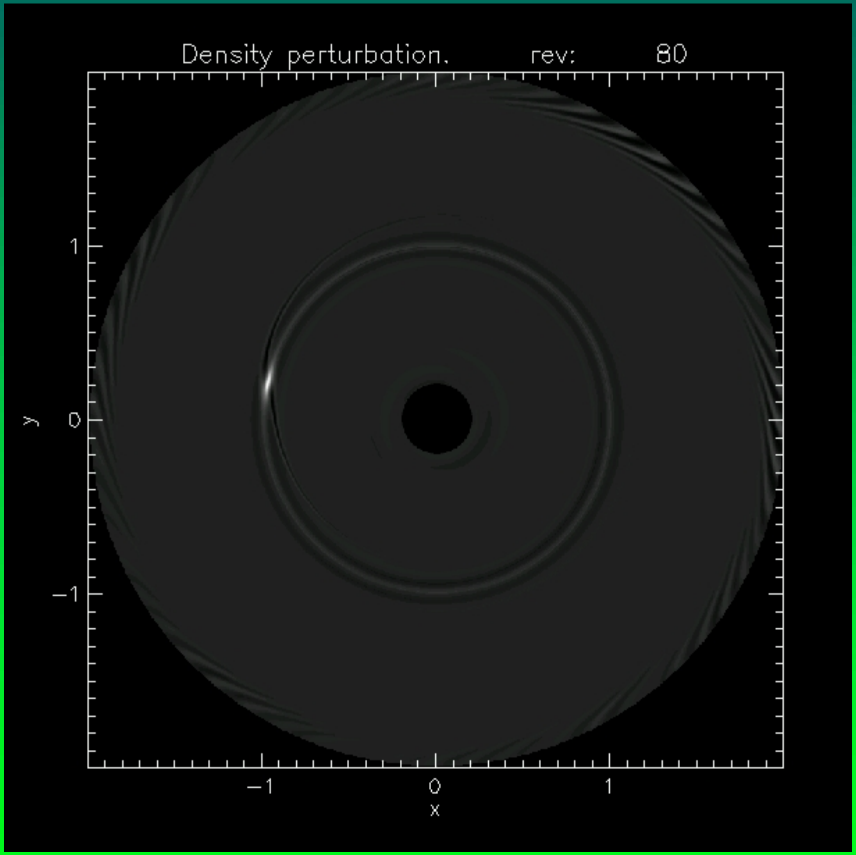




Vortex Stability

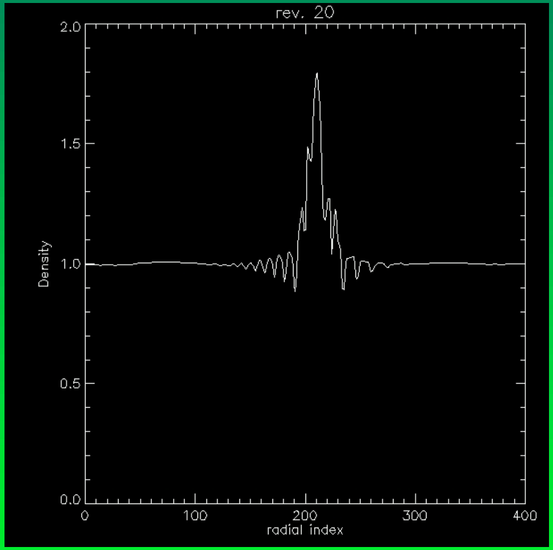
***ε*=0.5 *a*=0.1 res: (2000x733)**

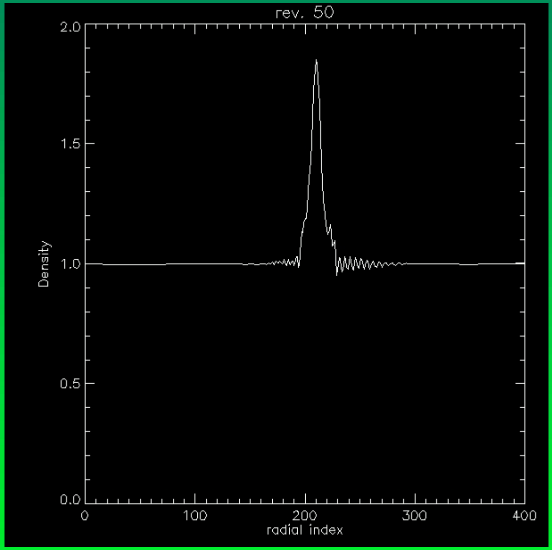


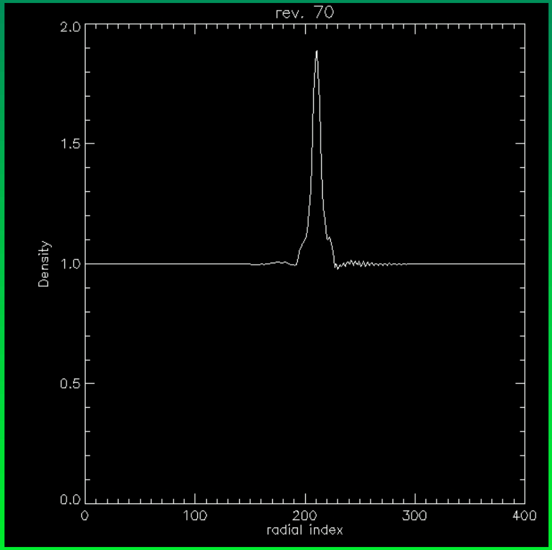


Vortex Stability

ε=0.5 a=0.1 res: (2000x733)







Adjustment Time

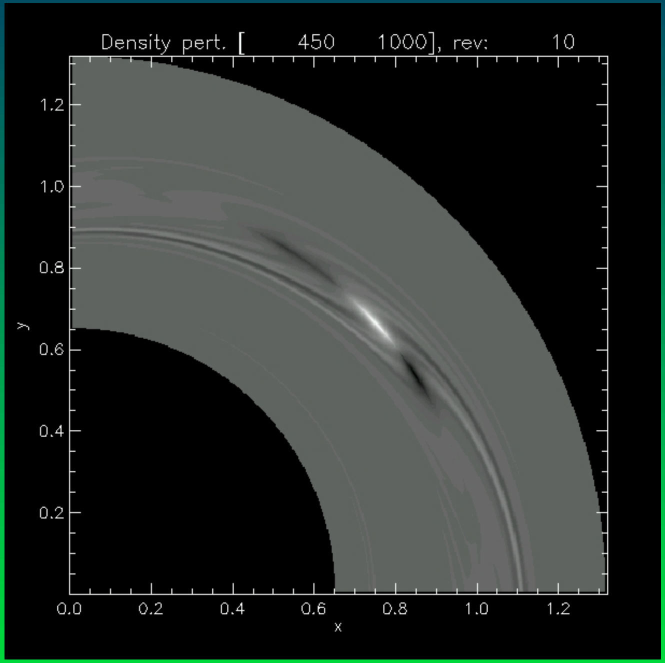
Depends on the initial imbalance:

$a=0.05$	3 revolutions
$a=0.1$	4 revolutions
$a=0.2$	6 revolutions

Does not depends on the wave speed

$Cs = 10^{-1}, 10^{-2}$

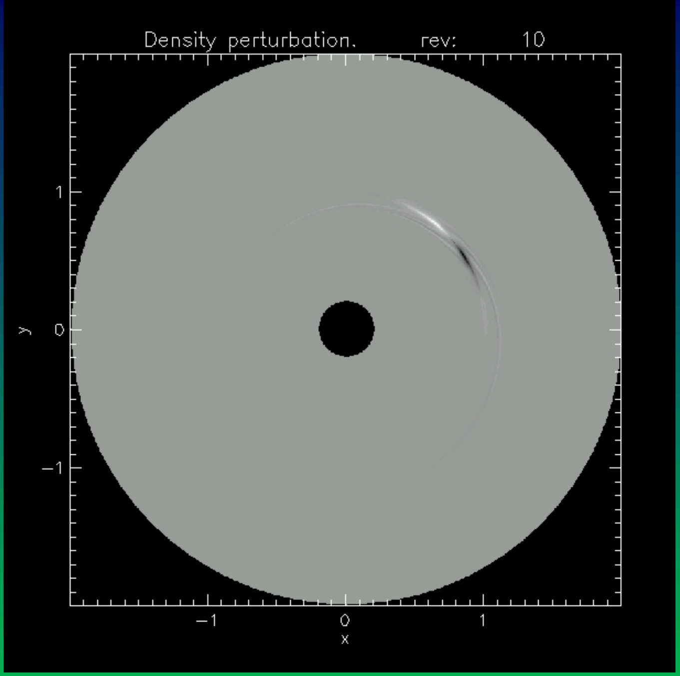
Resolution Study

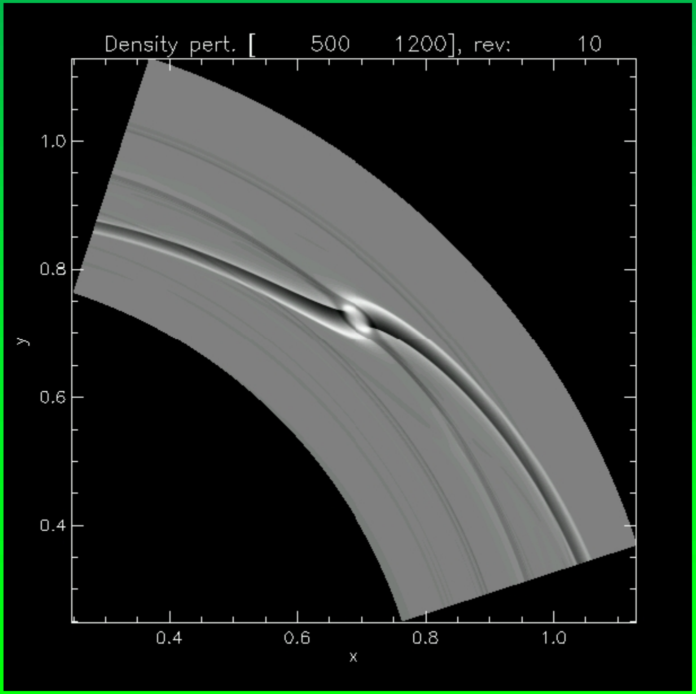


2000x733

4000x1466

8000x1559





Vortex Structure

Potential vorticity gradient is steepening, decreasing in size.

Size of the final vortex: ***$Cs = 10^{-2}$***

$a > 0.02$ ***$A \sim 0.02$***

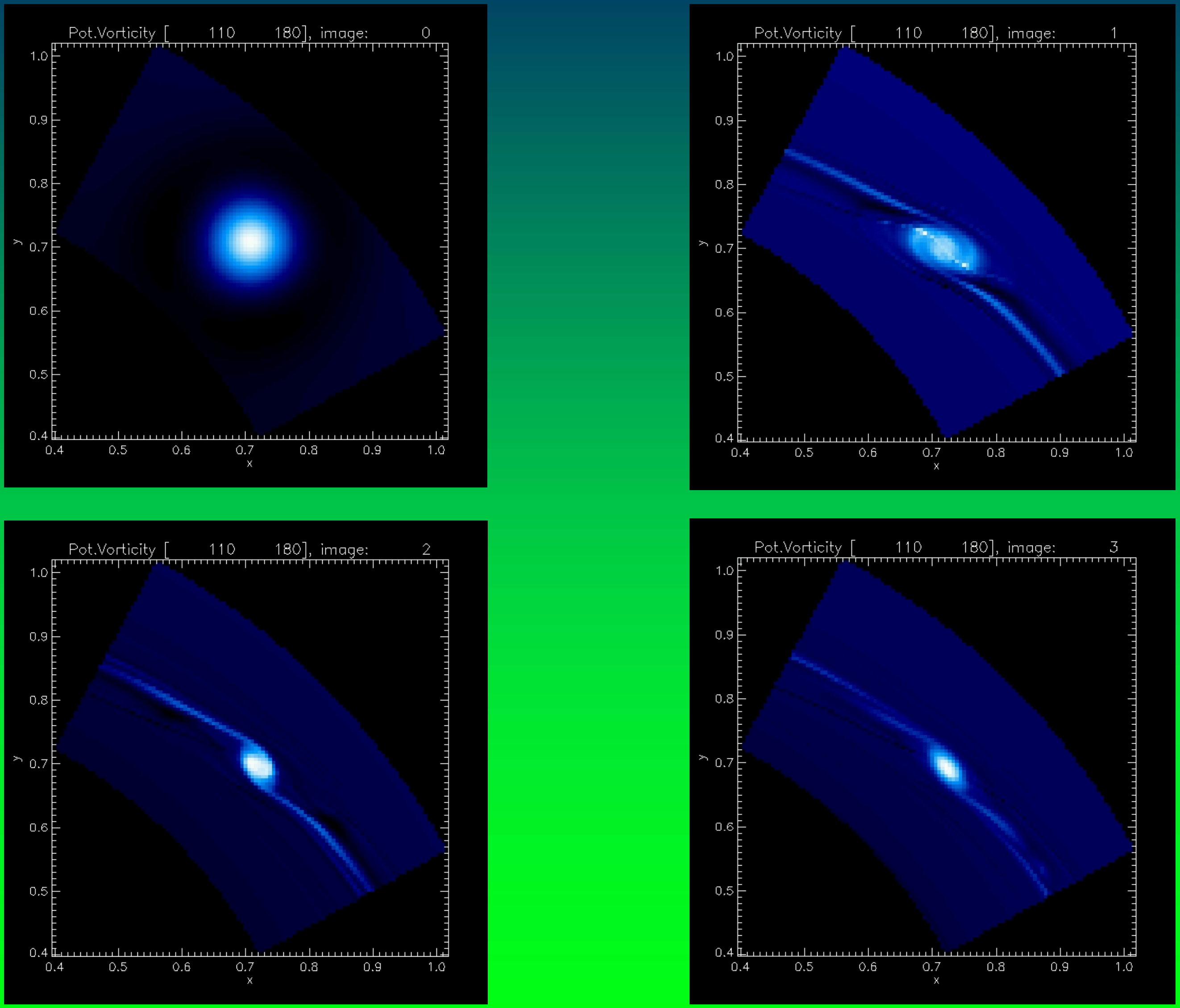
Double core vortex structure:

Adjustment of the potential vorticity ***(4)***

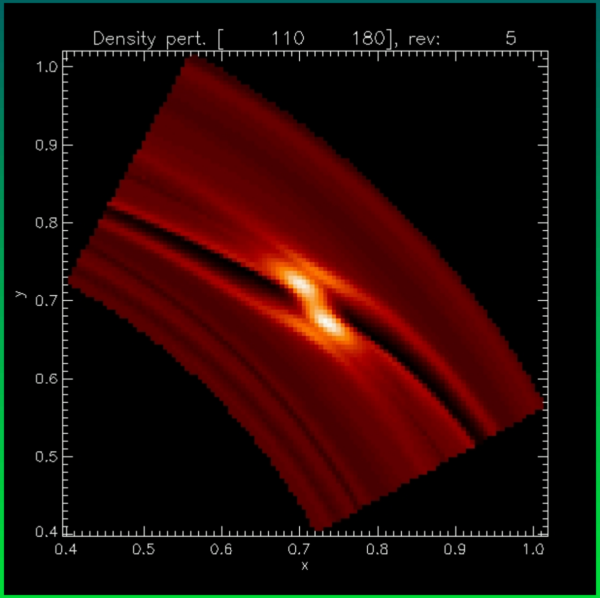
Adjustment of the mass ***(12)***

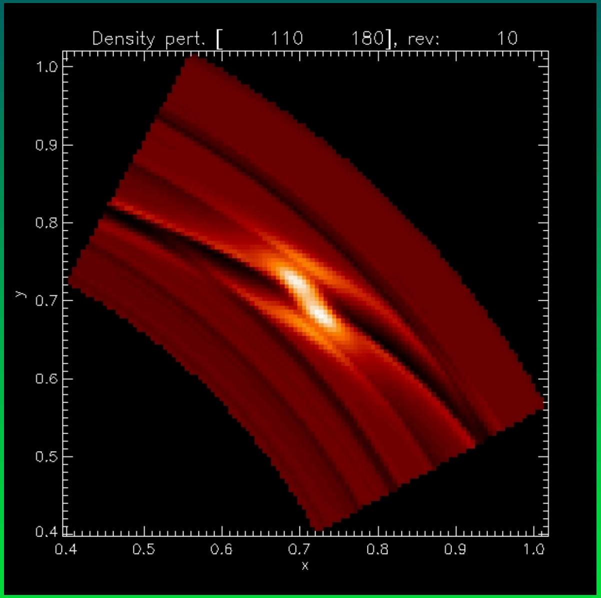
Different adjustment times scales?

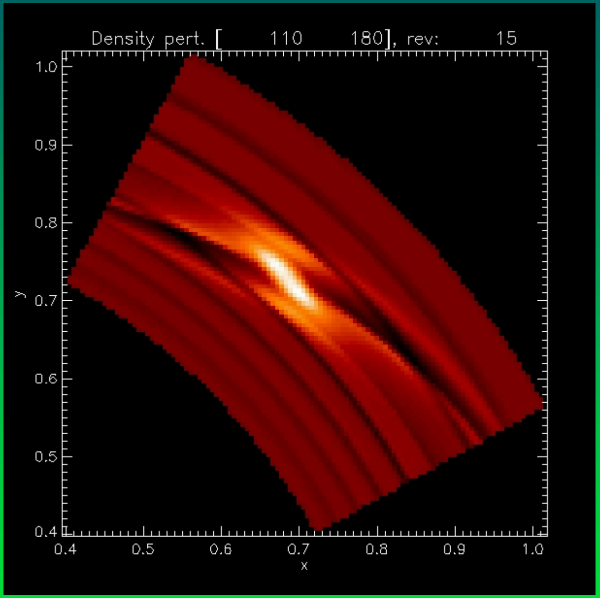
Adjustment of Potential Vorticity



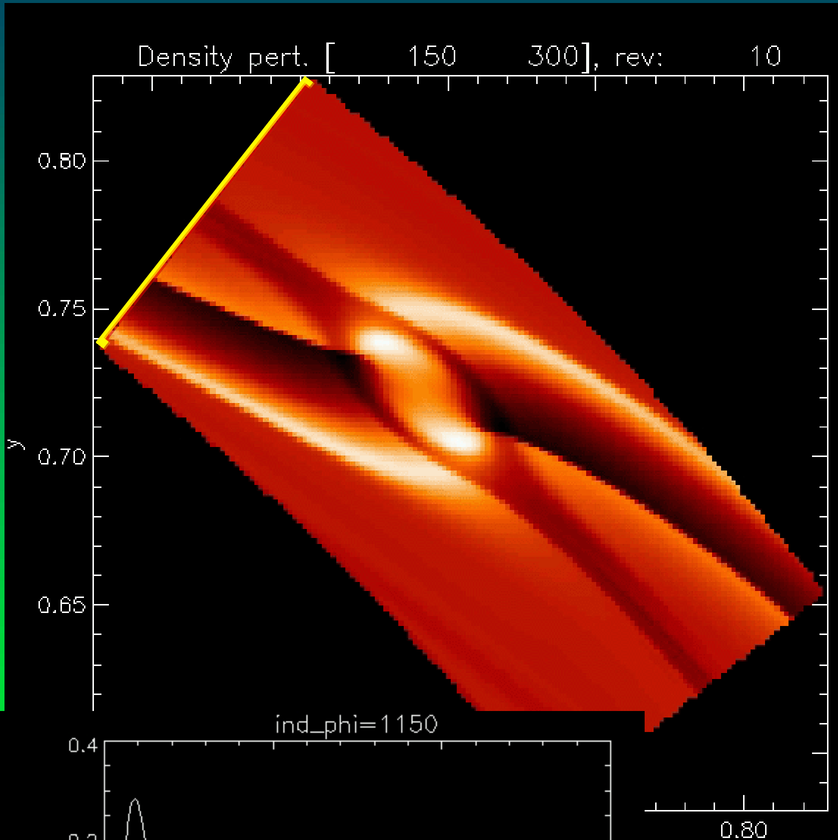
Double Core Vortex

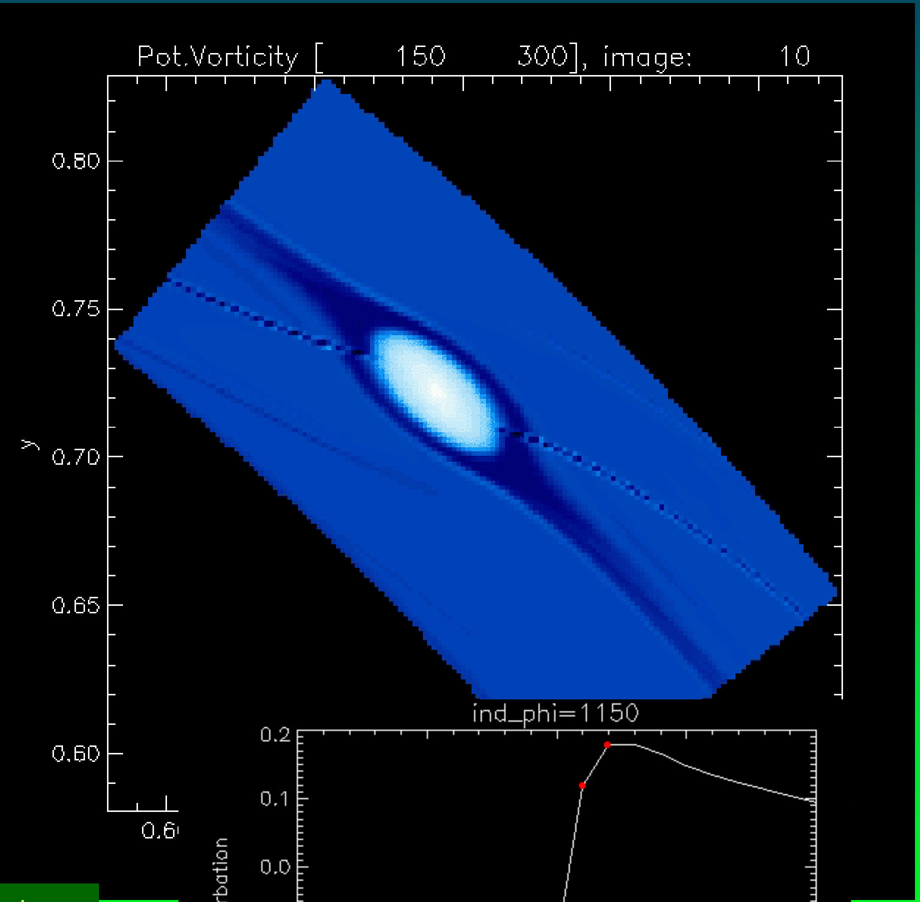


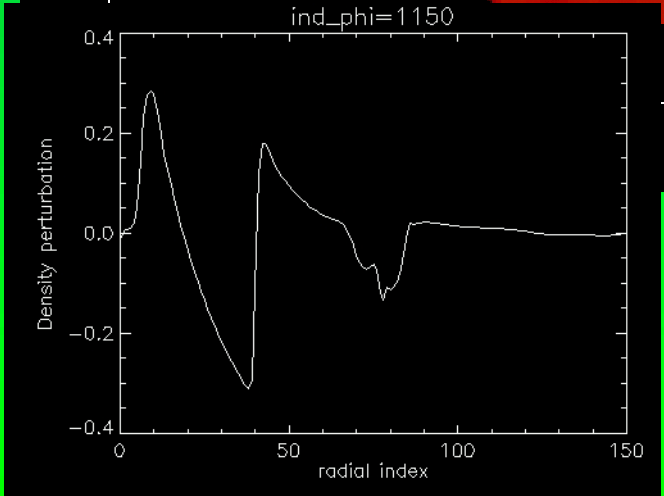




Wave Generation and Development: SHOCKS?







Parameters

Cs = 10e-2

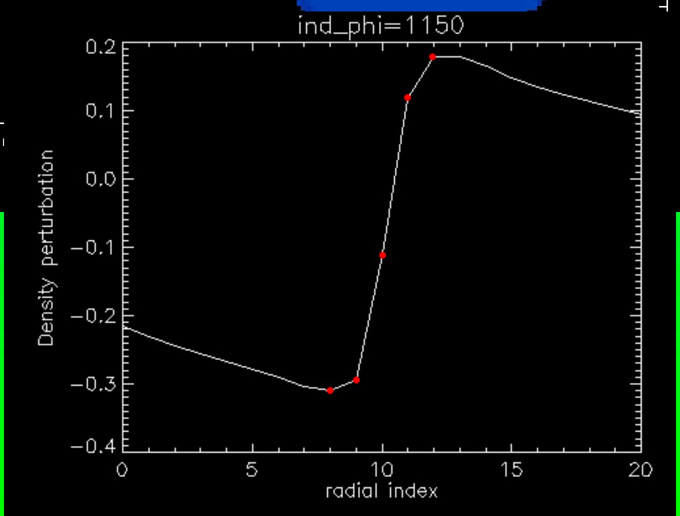
Eps = 0.5

Scale=0.05

8000x1559

Domain:

[0.5- 1.7]



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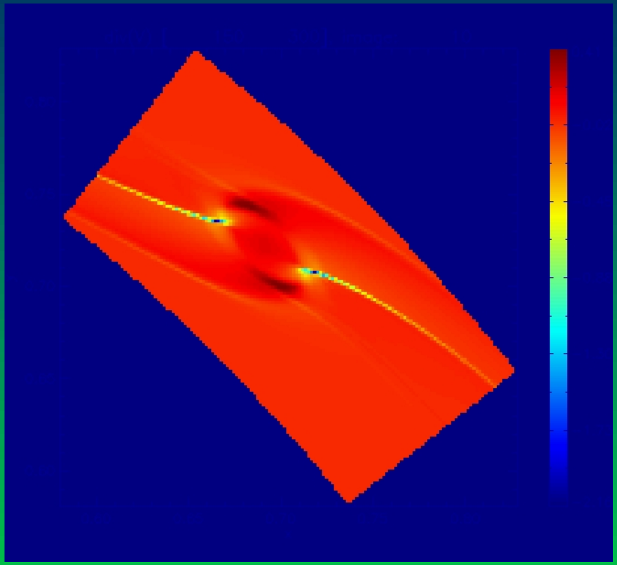

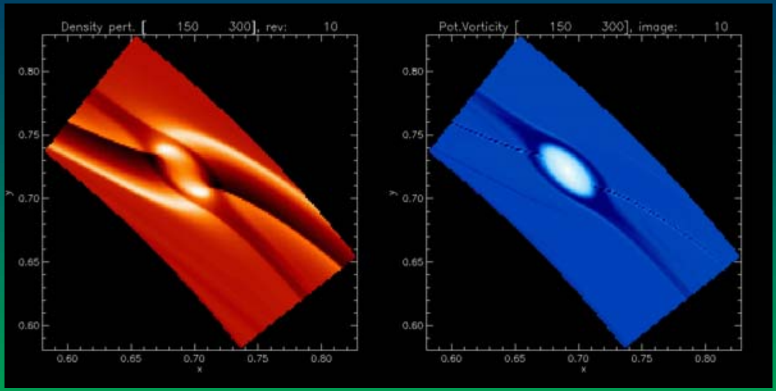

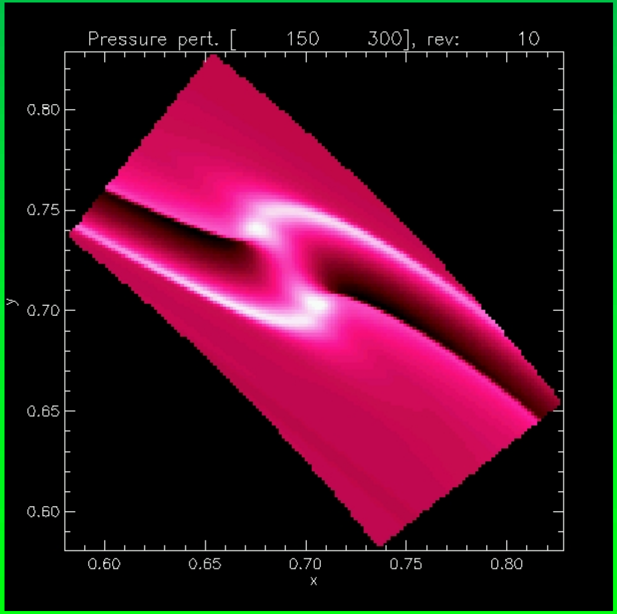

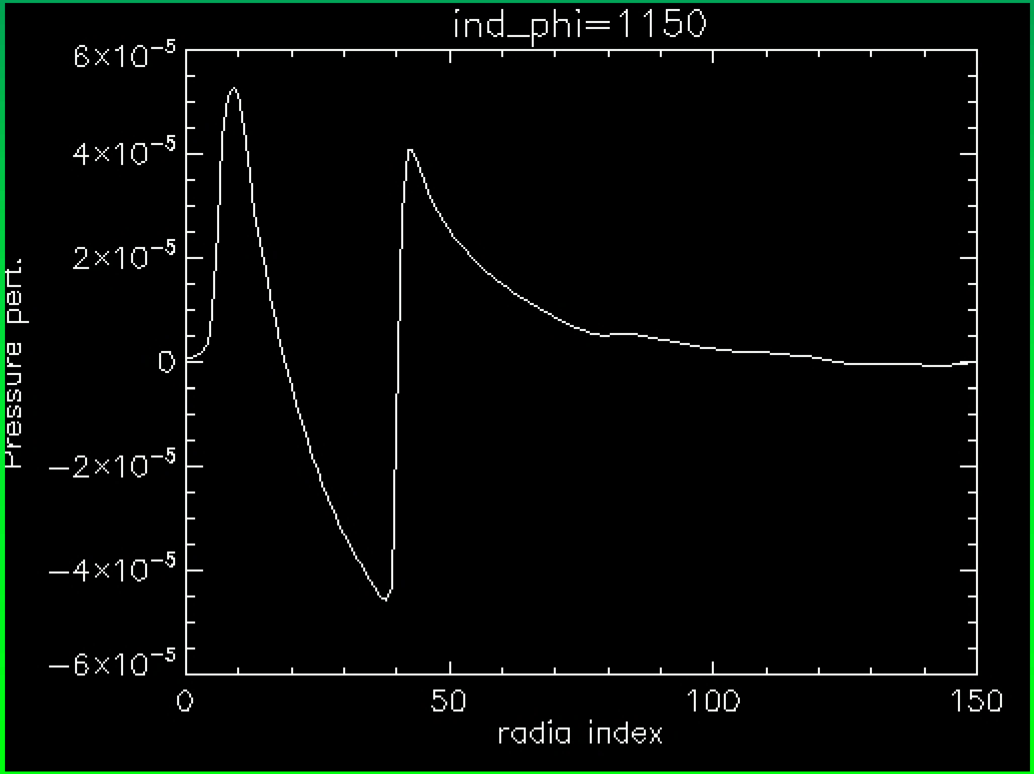

Summary

Stability

- Long lived anticyclonic vortices: stability 80+ revolutions
- Initial amplitude threshold values: 0.1,0.25

Nonlinear Adjustment

- Depends on the initial scale, not on the SD wave speed
- Adjustment time: Pot. Vort. – 4 revolutions (steepening)
  Density – 15 revolutions (double core)
- Size of the final vortex decreases to certain value if initial vortex is oversized (0.02)

Waves evolve into shocks (?)

Requirements on the minimal resolution for global simulations

Open Issues

- **Shocks? Do they dissipate faster then vortices?**
- **Momentum transport by waves/shocks**
- **Azimuthal diffusion – dense ring. Numerical?**
- **Slow drifting of the vortex. Numerical?**

  (v2=1.003, rev:50)