WAVES IN DUSTY, SOLAR, AND SPACE PLASMAS:

Linear dynamics of the solar convection zone - excitation of waves in unstably stratified shear flows

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Abstract

In this paper we report on the nonresonant conversion of convectively unstable linear gravity modes into acoustic oscillation modes in shear flows. The convectively unstable linear gravity modes can excite acoustic modes with similar wave-numbers. The frequencies of the excited oscillations may be qualitatively higher than the temporal variation scales of the source flow, while the frequency spectra of the generated oscillations should be intrinsically correlated to the velocity field of the source flow. We anticipate that this nonresonant phenomenon can significantly contribute to the production of sound waves in the solar convection zone.

INTRODUCTION

The excitation and propagation of waves are important for understanding the dynamics of the sun and stars. It is believed that most of the solar mechanical energy is accumulated in the turbulent motions in its convection zone. In the convection zone the gravitational stratification drives the convective instability providing the dynamical activity of this relatively thin region. The dynamics of the solar convection is studied to explain many observational features of the Sun. Notably, it is thought that the solar acoustic oscillations are excited by the turbulence in the convection zone [1-5].

Lighthill's ideas of aerodynamic sound generation form the basis of the theoretical investigation of the wave excitation in a hydrodynamic medium [6,7]. This theory of wave excitation by a free turbulence has been generalized for stratified fluids by Stein [8]. From a physical point of view, Lighthill's theory of wave generation employs the concept of stochastic excitation of oscillations (waves). In Lighthill's theory perturbations are described by an inhomogeneous wave equation, with linear terms forming the oscillatory part and the inhomogeneous terms standing for the source function. The source terms, which may be classified by their multipole order, are stochastically created by the turbulent perturbations. The amplifying effect of a sheared mean flow on the fluctuations of the Reynolds stress (nonlinear source term) and thus on the wave production has been noted by Lighthill [7]. However, this effect has not received further attention within the context of stochastic excitation.

Significant advances in the investigation of the dynamics of flows with velocity shear have been achieved together with the disclosure of specific features of shear flow phenomena [9,10]. Operators arising in the mathematical formalism of the canonical modal analysis in the study of the linear dynamics of shear flows are not self-adjoint. Consequently eigenmode interference introduces principal complications. The nonmodal approach has proved to be an alternative successful route for exploring the dynamics of shear flows. This approach employs the study of temporal evolution of the spatial Fourier harmonics of perturbations.

Impressive progress has been made by use of the nonmodal analysis (see e.g., [11-16]). This approach has led to the discovery of new channels of energy exchange between different modes in shear flows. Resonant phenomena of wave transformations have been studied in [17-23]. The nonresonant phenomenon of the conversion of vortices into acoustic waves has been described in [24]. The same mechanism is found to operate for magnetosonic [25] as well as for plasma Langmuir oscillations [21].

In this report we introduce a new dynamical source of acoustic waves in unstably stratified shear flows. Namely, the *linear* nonresonant conversion of convective into acoustic wave modes in a stratified shear flows. Convectively unstable exponentially growing buoyancy perturbations generate acoustic wave oscillations in presence of a sheared mean flow. We identify this linear conversion of modes in shear flows as a new excitation mechanism of the solar oscillations and waves. It differs in principle from the stochastic excitation mechanism and should significantly contribute to the process of acoustic wave generation in the solar convection zone.

Physical approach

The equations governing the dynamics of a compressible stratified flow are:

$$[\partial_t + (\mathbf{V}\nabla)]\,\rho + \rho(\nabla\mathbf{V}) = 0, \qquad (1.a)$$

$$[\partial_t + (\mathbf{V}\nabla)]\,\mathbf{V} = -\nabla P/\rho + \mathbf{g}, \qquad (1.b)$$

$$[\partial_t + (\mathbf{V}\nabla)]\,P = (\gamma P/\rho)\,[\partial_t + (\mathbf{V}\nabla)]\,\rho. \qquad (1.c)$$

We consider the hydrodynamic situation where a horizontal shear flow $\mathbf{V}_0 = (Ay, 0, 0)$ occurs in a vertically stratified medium $\mathbf{g} = (0, 0, -g)$. For simplicity we assume that $A = const$ and $g = const$. This yields the stratified equilibrium state:

$$P_0(z)/P_0(0) = \rho_0(z)/\rho_0(0) = \exp(-zk_H), \qquad (2)$$

where $k_H \equiv \gamma g / c_s^2$ and $c_s^2 \equiv \gamma P_0/\rho_0$. We introduce the linear perturbations in the following way:

$$\mathbf{V} = \mathbf{V}_0 + \mathbf{V}' \rho_0(0)/\rho_0(z), \quad P = P_0 + P', \quad \rho = \rho_0 + \rho'. \tag{3}$$

Here the velocity perturbations are normalized to exclude the exponential height dependence due to the vertical stratification of the background flow. We use the Cowling approximation [26] and neglect the perturbations of the gravitational acceleration. Following the standard method of nonmodal analysis (see [27] for a rigorous mathematical interpretation) we introduce the spatial Fourier harmonics (SFH) of the perturbations with time dependent phases:

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{k}(t), t) \exp(\mathrm{i}k_x x + \mathrm{i}k_y(t) y + \mathrm{i}\tilde{k}_z z), \tag{4.a}$$

$$k_y(t) = k_y(0) - A k_x t, \tag{4.b}$$

where $\tilde{k}_z \equiv k_z + \mathrm{i}k_H/2$. For compactness of notation we introduce the generalized vector of perturbations and their SFHs as follows: $\Psi \equiv (\mathbf{V}', p', \rho')$ and $\psi \equiv (\mathbf{u}, p, \rho)$. To avoid complex coefficients in the dynamical equations, we construct the normalized entropy and vertical velocity perturbation SFHs in the following way:

$$s \equiv (\mathrm{i}c_s^2 \tilde{k}_z^*/g - 1)(p - c_s^2 \rho)/(\gamma - 1), \tag{5.a}$$

$$v \equiv (c_s^2 \tilde{k}_z^* + \mathrm{i}g) u_z, \tag{5.b}$$

where $\tilde{k}_z^* = k_z - \mathrm{i}k_H/2$. From Eqs. (1-5) we obtain, by the use of straightforward manipulations, the following set of differential equations that govern the SFH of the linear perturbations in stratified shear flow:

$$\dot{p}(t) = c_s^2 (k_x u_x + k_y(t) u_y) + v, \tag{6.a}$$

$$\dot{u}_x(t) = -A u_y - k_x p, \tag{6.b}$$

$$\dot{u}_y(t) = -k_y(t) p, \tag{6.c}$$

$$\dot{v}(t) = (N_{\mathrm{B}}^2 - c_s^2 \bar{k}_z^2) p - N_{\mathrm{B}}^2 s, \tag{6.d}$$

$$\dot{s}(t) = v. \tag{6.e}$$

N_{B}^2 is the square of the frequency of the Brunt-Väisälä: $N_{\mathrm{B}}^2 \equiv g k_H (\gamma - 1)/\gamma$ and $\bar{k}_z^2 = |\tilde{k}_z|^2 = k_z^2 + k_H^2/4$. In an unstably stratified flow negative buoyancy ($N_{\mathrm{B}}^2 < 0$) requires that the adiabatic index $\gamma < 1$. Such an effective value may be assigned to this parameter under a certain thermodynamic approach [28]. However, in Eqs (6.a-e) we retain only N_{B}^2 and argue that these equations are more general than the underling γ prescription.

Further we note that vorticity is conserved in the wave-number space: $I = k_x u_y - k_y(t) u_x - (A/c_s^2)(p - s)$. The spectral energy of the perturbations can be defined as follows:

$$E = \rho_0/(2c_s^2) \left(E_{\mathrm{K}} + E_{\mathrm{P}} + E_{\mathrm{T}} \right), \tag{7.a}$$

$$E_{\rm K} = c_s^2(u_x^2 + u_y^2) + v^2/(c_s^2\bar{k}_z^2 - N_{\rm B}^2), \qquad (7.b)$$

$$E_{\rm P} = p^2, \qquad E_{\rm T} = N_{\rm B}^2 s^2/(c_s^2\bar{k}_z^2 - N_{\rm B}^2). \qquad (7.c, d)$$

where $E_{\rm K}$, $E_{\rm P}$ and $E_{\rm T}$ correspond to the kinetic, elastic and thermobaric energies of the perturbations, respectively. Formally the perturbation energy is conserved in the shearless limit: $\dot{E} = Ac_s^2 u_x u_y$. The instability of the convective eddies corresponds to a negative value of the thermobaric energy.

Linear modes in the shearless limit

The linear modes may be classified explicitly in the shearless limit ($A = 0$). In this case the full Fourier expansion of the linear perturbations $\Psi({\rm r}, t) \propto \tilde{\psi}({\rm k}, \omega)$ yields the dispersion equation:

$$\omega(\omega^4 - c_s^2k^2\omega^2 + N_{\rm B}^2c_s^2k_\perp^2) = 0, \qquad (8)$$

where $k_\perp^2 \equiv k_x^2 + k_y^2$ and $k^2 = k_\perp^2 + \bar{k}_z^2$. The solutions of Eq. (8) describe the stability and characteristic temporal variation scales of the existing modes:

$$\omega_{\rm v} = 0, \qquad (9.a)$$

$$\omega_{s,c}^2 = \frac{1}{2}c_s^2k^2\left\{1 \pm \left(1 - \frac{4N_{\rm B}^2k_\perp^2}{c_s^2\bar{k}^4}\right)^{1/2}\right\}, \qquad (9.b)$$

where the subscripts v, s, c define the frequencies of the vortex, acoustic and convective modes, respectively. In an unstably stratified flow, i. e., when $N_{\rm B}^2 < 0$, $\mathrm{i}\omega_c$ defines the growth rate of the buoyancy perturbations.

Obviously the $I = constant$ law demonstrates the existence of the stationary ($\omega = 0$) vortex mode in the linear spectum. The conserved vorticity I may be considered as the vortex mode measure. The physical eigenfunctions of the acoustic $\Phi_s(t)$ and convective $\Phi_c(t)$ modes may be rigorously defined in this limit:

$$\Phi_s(t) \equiv p(t) + N_{\rm B}^2\frac{\Omega_s^2 - \omega_s^2}{\Omega_c^4}\left(s(t) - \frac{\bar{k}_z^2}{k^2}p(t)\right) \qquad (10.a)$$

$$\Phi_c(t) \equiv s(t) - \frac{\bar{k}_z^2}{k^2}p(t) - \frac{\Omega_c^2 - \omega_c^2}{N_B^2}p(t) \qquad (10.b)$$

where $\Omega_s^2 \equiv c_s^2k^2$ and $\Omega_c^2 \equiv N_{\rm B}^2k_\perp^2/k^2$. Hence the equations governing the dynamics of the perturbations of the different modes may be decoupled as follows:

$$\ddot{\Phi}_s(t) + \omega_s^2\Phi_s(t) = 0, \qquad (11.a)$$

$$\ddot{\Phi}_c(t) + \omega_c^2\Phi_c(t) = 0. \qquad (11.b)$$

Starting from this simple situation we study the velocity shear effects on the perturbation modes.

Effects of a sheared flow

To study the effects of the velocity shear on the linear modes we introduce the small-scale approximation: $k_z^2 \gg k_H^2$. This approximation strongly simplifies the mathematical formulation and is justified for the following two reasons. Firstly, our analysis needs constant vertical gravity, an assumption that may be adopted for perturbations with vertical height scales shorter than the stratification scale. Secondly, this approximation is necessary for our assumption of a constant linear shear of the flow velocity, especially in the turbulent flows. Using Eq. (7) this approximation may be represented by the following condition: $c_s^2 k_z^2 \gg N_{\rm B}^2$. In terms of the frequencies it yields $(\Omega_s^2 - \omega_s^2) \approx (\Omega_c^2 - \omega_c^2) \approx 0$, which strongly simplifies the characteristic physical quantities of the perturbation modes: $\Phi_s(t) \approx p(t)$ and $\Phi_c(t) \approx (s(t) - \bar{k}_z^2 p(t)/k^2(t))$. To analyze the dynamics of acoustic oscillations in the shear flow we rewrite Eqs. (6.a-e) in the form of coupled second order differential equations for the variables $p(t)$ and $y(t)$:

$$\ddot{p}(t) + f(t)\dot{p}(t) + \Omega_1^2(t)p(t) = \lambda_1(t)\dot{y}(t) + \lambda_2(t)y(t), \tag{12.a}$$

$$\ddot{y}(t) + \Omega_2^2(t)y(t) = 0, \tag{12.b}$$

where the convection variable $y(t)$ is introduced as follows:

$$y(t) \equiv \frac{k_\perp(t)}{k(t)} \left(s(t) - \frac{\bar{k}_z^2}{k^2(t)} p(t) \right) \tag{13}$$

and

$$\Omega_1^2(t) = c_s^2 k^2(t) + 2A^2 \frac{k_x^2}{k^2(t)} - 4A^2 \frac{k_x^2 k_y^2(t) \bar{k}_z^2}{k_\perp^2(t) k^4(t)}, \tag{14.a}$$

$$\Omega_2^2(t) = N_{\rm B}^2 \frac{k_\perp^2(t)}{k^2(t)} + 2A^2 \frac{k_x^2 k_z^2}{k_\perp^4(t) k^4(t)} \left[3k_\perp^2(t)k^2(t) - 4k_y^2(t)k_\perp^2(t) - k_y^2(t)\bar{k}_z^2 \right], \tag{14.b}$$

$$f(t) = 2A \frac{k_x k_y(t)}{k^2(t)}, \tag{14.c}$$

$$\lambda_1(t) = -2A \frac{k_x k_y(t)}{k_\perp(t) k(t)}, \tag{14.d}$$

$$\lambda_2(t) = -2A^2 \frac{k_x^2 k_\perp(t)}{k^3(t)} \left(1 - \frac{k_y^2(t) \bar{k}_z^2}{k_\perp^4(t)} \right). \tag{14.e}$$

In deriving Eqs. (12.a-b) we have used the following two simplifications. Firstly, we have retained only the terms describing the effect of the buoyancy perturbations on the acoustic waves, and we have neglected the effect of the acoustic pressure perturbations on the evolution (exponential amplification) of the buoyancy perturbations in the right hand side (rhs) of Eq. (12.b). Secondly, we have neglected the source terms in the rhs of the two dynamical equations that describe the shear induced coupling between the vortex and acoustic wave modes (in Eq. 12.a) and vortex and buoyancy modes (in Eq. 12.b). In fact, the coupling of the vortex and acoustic wave is a process that has been studied to reveal the mean flow shear induced nonresonant mode conversion phenomenon in [24]. However, in the present case, the source terms of the acoustic waves that are proportional to the vortex mode measure, conserved quantity I, are dominated by the source terms, associated with the exponentially amplifying convective modes: $y(t)$ and $\dot{y}(t)$. It should be emphasized that the present approach is justified only for a convectively unstable medium with $N_{\rm B}^2 < 0$, so that the buoyancy modes undergo exponential amplification in the linear regime.

The dynamics of the acoustic waves in the absence of the buoyancy perturbations is described by the homogeneous part of Eq. (12.a). The acoustic wave frequency and amplitude variations are described by the parameters $\Omega_1^2(t)$ and $f(t)$ (see [19] for a detailed study). The dynamics of the convective mode is described by Eq. (12.b). Eq. (16.b) shows the transient stabilization effect of the sheared mean flow in an unstably stratified medium. The stabilization occurs at times, when $|k_y(t)/k_x| < 1$ and reaches its maximum at $t = t^*$, when $k_y(t^*) = 0$ (see Eq. 16.b).

The terms $\lambda_1(t)\dot{y}(t)$ and $\lambda_2(t)y(t)$ in the rhs of Eq. (14.a) describe the coupling between the convective and acoustic waves modes. The shear flow origin of these source terms is obvious from Eqs. (16.d,e). Hence, Eqs. (14.a,b) describe the mean flow shear induced buoyancy – acoustic wave mode conversion in a convectively unstable medium. Some specific features of this phenomenon are due to its linear nature; SFH of the exponentially growing buoyancy perturbations are able to generate SFH of the acoustic waves with the same wave-numbers. The amplitude of the excited wave mode depends on the values of the source terms $\lambda_1(t)$ and $\lambda_2(t)$. So, convective modes with $k_x = 0$ can not generate acoustic waves at all ($\lambda_1 = \lambda_2 = 0$). While maximal efficiency of the mode conversion phenomenon should occur at $k_z = 0$, or in a realistic physical approximation (see Eq. 12) at $k_z^2 \geq N_{\rm B}^2/c_s^2$. Naturally, acoustic wave emission from convection should generally increase when the mean flow shear parameter A increases.

We numerically analyze Eqs. (6.a-e) to verify the analytical results obtained from the approximate equations (12.a,b). We select the initial perturbations in a specific manner, which enables us to excite the convective and acoustic wave modes individually at the initial moment of time. It appears that exponentially growing buoyancy perturbations instantly excite the acoustic wave mode harmonics at a given point in time, when the perturbation wave-number along the flow velocity shear is zero: $t = t^*$, $k_y(t^*) = 0$. The generation of acoustic waves is clearly traced from the pressure variation, as well as the compression of the perturbations. Numerical analysis shows that the efficiency of this mode conversion phenomenon increases with the flow shear parameter.

DISCUSSION AND CONCLUSIONS

We have presented a study of compressible convection in shear flows. In particular we have focused on linear small-scale perturbations in unstably stratified flows with constant shear of velocity. The linear character of the system enables us to identify the perturbation modes and to study their dynamics individually. We find a mode conversion that originates from the velocity shear of the flow: exponentially growing perturbations of convection are able to excite acoustic waves. This process offers a novel approach to the hydrodynamic problem of the acoustic wave generation.

This wave excitation phenomenon can be important for the acoustic oscillations of the sun. Being responsible for the wave generation in high shear regions of a stratified turbulent flow, this nonresonant phenomenon can contribute to the production of sound in the solar convection zone. Moreover, the process of the wave excitation should be triggered by a weak vertical magnetic field. In this case we anticipate the production of high frequency compressional MHD waves. The latter process will considerably increase the extraction of the mechanical energy of the convection by waves.

Specific to this phenomenon is that perturbations of buoyancy are able to excite acoustic waves with similar wave-numbers. This property makes it clearly distinct from stochastic excitation, where the generated frequencies are similar to the life-times of the source perturbations. In contrast, frequencies of the oscillations generated by the mean flow velocity shear induced mode conversion may be qualitatively higher than the temporal variation scales of the perturbations in the source flow of a compressible convection. The frequency spectrum of the excited acoustic waves should be intrinsically correlated to the velocity field of the turbulent source flow. Shear flow induced wave excitation in stratified flows offers a natural explanation of the fact, that the solar acoustic oscillation are mainly excited in the high shear regions of the convection, intergranular dark lanes [29]. It also explains the puzzling wave-number dependence of the observed mode energies at fixed frequencies (see [5] and references therein). A detailed comparison with observational data requires a more realistic physical model. The simplicity of our model is used to demonstrate the basic features of this excitation phenomenon.

Finally we note that in the present formalism we have focused on the waves with frequencies higher than the characteristic cut-off frequency for the acoustic waves in the convection zone. Shear flow initiates the qualitative change of the temporal variation scales of perturbations and the excitation of the waves that are not trapped in the convective envelope. Hence, this mode conversion presents a new significant contribution into the channel of energy transfer from the dynamically active interior to the atmosphere of the Sun.

ACKNOWLEDGMENTS

A. G. Tevzadze would like to acknowledge the financial support as "bursaal" of the "FWO Vlaanderen", project G.0335.98. This work was supported in part by the INTAS grant GE97-0504.

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