LINEAR SOURCES OF ACOUSTIC WAVES IN THE SHEAR FLOWS OF SOLAR CONVECTION

A. G. Tevzadze[1], G. D. Chagelishvili[1], and M. Goossens[2]

[1]*Abastumani Astrophysical Observatory, 2a Al. Kazbegi Ave. Tbilisi, 0160 Georgia*
[2]*Center for Plasma Astrophysics, K.U.Leuven, Celestijnenlaan 200B, Leuven 3001, Belgium*

ABSTRACT

We study the linear phenomenon of the wave excitation in the convectively unstable shear flows. For this purpose we use local parallel shear flow approach and analyze the dynamics of perturbations in the horizontal flow with vertical stratification. We identify the phenomenon of the linear mode conversion as the source of acoustic waves and study the process using non-modal approach. Perturbations of the unstable buoyancy mode are able to excite acoustic wave mode with similar wave-numbers in the linear regime. We use the non-modal analysis as the optimal tool for the description of the transient process of mode coupling and reveal the sources of acoustic waves in the form of the pressure modified entropy perturbations.

The mode conversion concept employed to analyze the wave excitation differs from the standard Lighthill mechanism and is complementary for the estimate of the acoustic production in shear flows. Improved estimate of the acoustic wave excitation rates can be obtained in the flows with higher shear rates. The wave excitation rate strongly reduces with the decrease of the Mach number of the flow. The variations in the wave excitation rate are analyzed numerically.

We discuss the modification of acoustic production in the solar convection zone due to the kinematic inhomogeneities of the flow. In this situation acoustic waves can be excited by both, stable vertex and marginally unstable buoyancy mode perturbations with similar length-scales. Since the solar convection drives the main energy into the buoyancy (thermal) perturbations we neglect the vortex induced acoustic production and analyze the convective perturbations with zero potential vorticity.

We discuss the flow parameters where the non-modal wave production can be important ingredient of the acoustic power spectrum. We describe the characteristic features of the linearly excited acoustic waves which makes their contribution into the overall acoustic power spectrum clearly distinctive from the acoustic production due to the standard stochastic excitation. We discuss the helioseismic implications of the described phenomenon.

Key words: The Sun: convection, acoustic waves.

1. INTRODUCTION

The subject of the genesis of solar oscillations is an important problem in solar. Several different mechanisms have been offered to explain the excitation of the solar p-mode oscillations in the convection zone. Lighthill's (1952) classical problem of the turbulent sound production has been extended for the convective turbulence by Stein (1967) who has also estimated the acoustic energy flux from the solar convection zone due to the stochastic excitation of oscillations. Several authors have developed this approach later (see e.g. Goldreich and Kumar 1990) which employs the acoustic sources of the stochastic nature resulting from the turbulent fluctuations of the Reynolds stress in the upper layers of the convection zone. Among the alternative mechanisms is the thermal overstability of the acoustic oscillations in the convection zone that has been proposed by Balmforth (1992) as a major source of acoustic oscillations.

In present paper we focus on the acoustic production due to the velocity shear of the background flow, persistent to the flows in the solar convection zone. We employ the recent progress in the understanding of the shear flow phenomena and use these results in the astrophysical context.

Here we focus on sources rather than the observational frequencies and the global characteristics of the solar oscillations. We found that the linear perturbations of the convection are able to excite the acoustic waves due to the velocity shear induced mode conversion phenomenon. Hence we introduce the novel channel of the wave excitation in the solar convection zone. Interplay of the kinematic inhomogeneities and the unstable stratification on the top of the convection zone may produce a significant acoustic levels due to the presented linear mechanism.

Proceedings of the 11th European Solar Physics Meeting - The Dynamic Sun: Challenges for Theory and Observations, 11-16 September 2005 (ESA SP-596, December 2005)

2. FORMALISM

We consider a parallel shear flow $\mathbf{V}_0 = (Ay, 0, 0)$ in the presence of a constant vertical gravity acceleration: $\mathbf{g} = (0, 0, -g)$ with $g = constant$. We consider equilibrium flow:

$$\frac{P_0(z)}{P_0(0)} = \frac{\rho_0(z)}{\rho_0(0)} = \exp(-k_H z), \tag{1}$$

where $k_H \equiv \gamma g/c_s^2$ is the vertical stratification scale and $c_s^2 \equiv \gamma P_0/\rho_0$ is the sound speed. The fact that an equilibrium solution exists does not means the existence of the stable state. The flow may be thermally stable or unstable depending on the thermodynamic parameters of the system (in the present model – depending on the parameter γ). Indeed, the convectively unstable flow is the subject of interest in the present case. The linear perturbations may be introduced in the following way:

$$\mathbf{V} = \mathbf{V}_0 + \frac{\rho_0(0)}{\rho_0(z)}\mathbf{V}', \;\; P = P_0 + P', \;\; \rho = \rho_0 + \rho'. \tag{2}$$

Here the velocity perturbations are normalized to exclude the exponential height dependence due to the vertical stratification of the background flow. We use the Cowling approximation and neglect perturbations of the gravitational acceleration.

Following the standard method of the non-modal analysis we introduce the spatial Fourier harmonics (SFH) of perturbations with time dependent phases:

$$\left\{ \begin{array}{c} p'(\mathbf{r},t) \\ \rho'(\mathbf{r},t) \\ \mathbf{V}'(\mathbf{r},t) \end{array} \right\} = \left\{ \begin{array}{c} p(\mathbf{k},t) \\ \rho(\mathbf{k},t) \\ \mathbf{u}'(\mathbf{k},t) \end{array} \right\} \exp(\mathrm{i}k_x x + \mathrm{i}k_y(t)y + \mathrm{i}\tilde{k}_z z), \tag{3}$$

$$k_y(t) = k_y(0) - Ak_x t, \tag{4}$$

where $\tilde{k}_z \equiv k_z + \mathrm{i}k_H/2$. This complex wave-number is employed to re-scale the vertical axis in the stratified medium. To avoid complex coefficients in the dynamical equations, we construct the normalized entropy and vertical velocity perturbation SFHs in the following way:

$$s \equiv \left(\mathrm{i}c_s^2\tilde{k}_z^* - g\right)\frac{p - c_s^2\rho}{(\gamma - 1)g}, \tag{5}$$

$$v \equiv (c_s^2\tilde{k}_z^* + \mathrm{i}g)u_z, \tag{6}$$

where $\tilde{k}_z^* = k_z - \mathrm{i}k_H/2$. Hence, equations governing the dynamics of linear perturbations in the considered model are the following:

$$\frac{\mathrm{d}}{\mathrm{d}t}p(t) = c_s^2(k_x u_x + k_y(t)u_y) + v, \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}u_x(t) = -Au_y - k_x p, \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}u_y(t) = -k_y(t)p, \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}v(t) = (N_{\mathrm{B}}^2 - c_s^2\bar{k}_z^2)p - N_{\mathrm{B}}^2 s, \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}s(t) = v. \tag{11}$$

N_{B}^2 is the square of the Brunt-Väisälä frequency: $N_{\mathrm{B}}^2 \equiv gk_H(\gamma - 1)/\gamma$ and $\bar{k}_z^2 = |\tilde{k}_z|^2 = k_z^2 + k_H^2/4$. In order to model convective instability we adopt effective γ model introduced in Ryu & Goodman 1992. The essence of this approximation is that we can use the artificial gamma parameter instead of adiabatic factor when the flow is heated uniformly and cooled according to a certain thermodynamic law. In this case the effective $\gamma < 1$, while the adiabatic index $\Gamma = 5/3$.

2.1. Eigenmodes

The mode frequencies can be obtained in the zero shear limit:

$$\omega_{\mathrm{v}} = 0,$$

$$\omega_s^2 = \frac{1}{2}c_s^2 k^2\left\{1 + \left(1 - \frac{4N_{\mathrm{B}}^2 k_\perp^2}{c_s^2\bar{k}^4}\right)^{1/2}\right\},$$

$$\omega_c^2 = \frac{1}{2}c_s^2 k^2\left\{1 - \left(1 - \frac{4N_{\mathrm{B}}^2 k_\perp^2}{c_s^2\bar{k}^4}\right)^{1/2}\right\},$$

Corresponding eigenfunctions may be written as:

$$\Phi_s(t) \equiv p(t) + N_B^2\frac{\Omega_s^2 - \omega_s^2}{\Omega_c^4}\psi(t),$$

$$\Phi_c(t) \equiv \psi(t) - \frac{(\Omega_c^2 - \omega_c^2)}{N_B^2}p(t),$$

were

$$\psi(t) \equiv \left(s(t) - \frac{\bar{k}_z^2}{k^2}p(t)\right),$$

and

$$\Omega_s^2 = c_s^2 k^2 - N_B^2\frac{k_\perp^2}{k^2},$$

$$\Omega_c^2 = N_B^2\frac{k_\perp^2}{k^2},$$

In uniform flow modes are not coupled:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Phi_s(t) + \omega_s^2\Phi_s(t) = 0,$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\Phi_c(t) + \omega_c^2\Phi_c(t) = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(k_x u_y - k_y u_x) = 0,$$

Velocity shear affects the modes and induces coupling of these modes and non-resonant mode coupling can occur (e.g., Chagelishvili et al. 1997). Hence, we consider the shear flow effect on the acoustic and buoyancy g-mode individually.

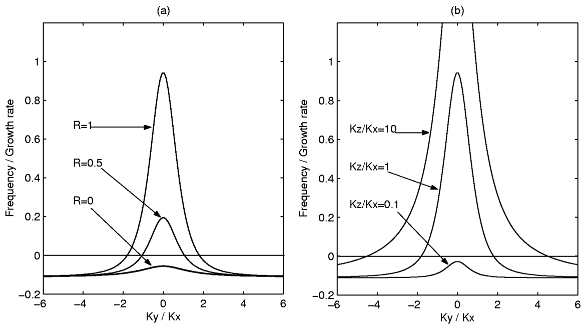

Figure 1. Frequency (growth rate) of the buoyancy perturbations $\Lambda(\mathbf{k})$ vs. ratio $k_y(t)/k_x$ in the case of thermally unstable flow $\gamma = 0.9$. Panel (a) shows the growth rate (when $\Lambda < 0$) and frequency (when $\Lambda > 0$) of the SFH of g-mode for different rates of velocity shear $R = 0,\ 0.5$ and 10 when $k_x = 10$ and $k_z = 10$. Panel (b) shows the same graphs at different wave-numbers $k_z/k_x = 0.1,\ 1$ and 10 when R=1.

2.2. Convection

To analyze the unstable g-mode dynamics we set eigenfunctions of vortex and acoustic waves to zero and derive the equation describing the dynamics of buoyancy perturbations using the following equation:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t) + \Lambda(t)y(t) = 0, \tag{12}$$

where

$$y(t) = \psi(t)\exp\left(\frac{1}{2}\int\beta_1(t)\mathrm{d}t\right), \tag{13}$$

and

$$\Lambda(t) = \beta_2(t) - \frac{1}{2}\left(\beta_1^2(t) + \frac{\mathrm{d}}{\mathrm{d}t}\beta_1(t)\right). \tag{14}$$

The stability of the buoyancy perturbations can be described by the function $\Lambda(t)$. To illustrate the velocity shear effect on the SFH of convective perturbations we plot $\Lambda(t)$ for different velocity shear rates R as well as for different wave-numbers on Fig. 1. It seems that the velocity effect may even change the sign of $\Lambda(t)$ and hence change the character of the evolution of perturbations from exponential growth to the stable evolution. As illustrated in graph 5.1.a the stabilization increases proportional to the shear parameter and is most profound in the vicinity of the point where $k_y(t) = 0$. In short, the velocity shear of the background flow exerts a stabilization effect on the convection depending on the velocity shear rate at particular wave-numbers transiently.

2.3. Acoustic waves

We can derive the equation governing the dynamics of acoustic waves in the local shear flow approximation,

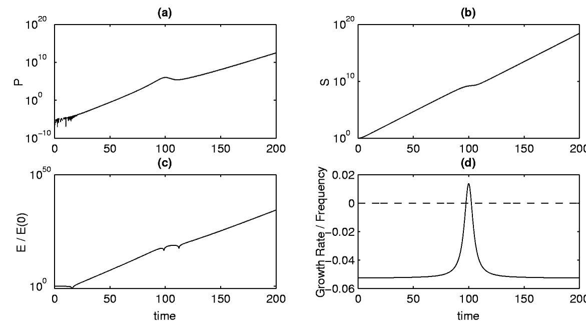

Figure 2. Evolution of a buoyancy perturbations in the convectively unstable medium: $\sigma = -0.055$ ($\gamma = 0.95$), $K_x = K_z = 10$, $K_y(0) = 200$ and $R = 0.2$. Pressure $P(\tau)$, Entropy $S(\tau)$ normalized total energy density $E(\tau)/E(0)$ and convective stability parameter $\Lambda(\tau)$ are shown on the a, b, c and d panels, respectively.

where $\omega_s \approx \Omega_s$:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}p(t) + \alpha_1(t)\frac{\mathrm{d}}{\mathrm{d}t}p(t) + \alpha_2(t)p(t) = = -\alpha_3(t)\frac{\mathrm{d}}{\mathrm{d}t}\psi(t) - \alpha_4(t)\psi(t), \tag{15}$$

where mode coupling coefficients are:

$$\alpha_3(t) \approx -2A\frac{k_x k_y(t)}{k_\perp(t)k(t)}, \tag{16}$$

$$\alpha_4(t) \approx -2A^2\frac{k_x^2 k_\perp(t)}{k^3(t)}\left(1 - \frac{k_y^2(t)\bar{k}_z^2}{k_\perp^4(t)}\right). \tag{17}$$

The acoustic perturbations are described by Eq. (15) which is the equation of the forced oscillator with the inhomogeneous terms are associated with the velocity shear and the convective mode perturbations. The acoustic waves in the absence of the buoyancy perturbations are described by the homogeneous part of this equation. The acoustic wave frequency and amplitude variations are described by the parameters $\alpha_1(t)$ and $\alpha_2(t)$.

The terms in the rhs of Eq. (15) describe the coupling between the convective and acoustic waves modes. This coupling implies that acoustic waves can be excited by the buoyancy perturbations. As we verify numerically (see Figs. 2-4), unstable g-mode perturbations are able to excite acoustic waves at a particular point in time, or in other words at particular wave-numbers depending on the initial wave-numbers of the SFH of perturbations and the velocity shear parameter. So, convective modes with zero streamwise wavenumbers ($k_x = 0$) are able to generate acoustic waves at all ($\alpha_3 = \alpha_4 = 0$ when $k_x = 0$). The maximal efficiency of mode conversion should occurs at $k_z = 0$, or in a realistic physical approximation at $k_z^2 \geq N_{\mathrm{B}}^2/c_s^2$. Acoustic wave emission from convection increases when the mean flow shear parameter A increases.

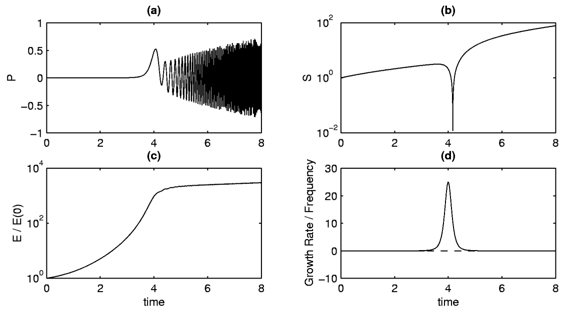

Figure 3. Evolution of a g-mode in the convectively unstable medium: $\sigma = -0.055$ ($\gamma = 0.95$), $K_x = K_z = 10$, $K_y(0) = 200$ and $R = 5$. Evolution of the pressure reveal the acoustic wave generation phenomenon. Generation of oscillations occurs at time $\tau = 4$, when wave-number along the flow shear is zero $K_y(4) = 0$.

3. DISCUSSION

We have presented a study of compressible convection in shear flows. In particular we have focused on linear small-scale perturbations in unstably stratified flows with constant shear of velocity. The linear character of the system enables us to identify the modes and to study their dynamics individually. We find a mode conversion that originates from the velocity shear of the flow: exponentially growing perturbations of convection are able to excite acoustic waves. In particular wave-numbers thermal perturbations (perturbations of buoyancy) feed the acoustic radiation. The generated oscillations are spatially correlated with the source flow. This process offers a novel approach to the hydrodynamic problem of the acoustic wave generation.

Specific to this phenomenon is that perturbations of buoyancy are able to excite acoustic waves with similar wave-numbers. This property makes it clearly distinct from stochastic excitation, where the generated frequencies are similar to the life-times of the source perturbations. In contrast, frequencies of oscillations generated by the mean flow velocity shear induced mode conversion may be qualitatively higher than the temporal variation scales of perturbations in the source flow of a compressible convection. The frequency spectrum of the excited acoustic waves should be intrinsically correlated to the velocity field of the turbulent source flow. Shear flow induced wave excitation in stratified flows offers a natural explanation of the fact, that the solar acoustic oscillation are mainly excited in the high shear regions of the convection, intergranular dark lanes (see Rimmele et al. 1995).

In the investigation of the observable acoustic sources of the sun Strous et al. 2000 have found that the excited acoustic power in the seismic events is linearly proportional to the convective velocity. This should indicate that acoustic waves are produced due to the linear mechanism such as the mode conversion and not the stochastic mechanism, where the dependence of the convective velocity

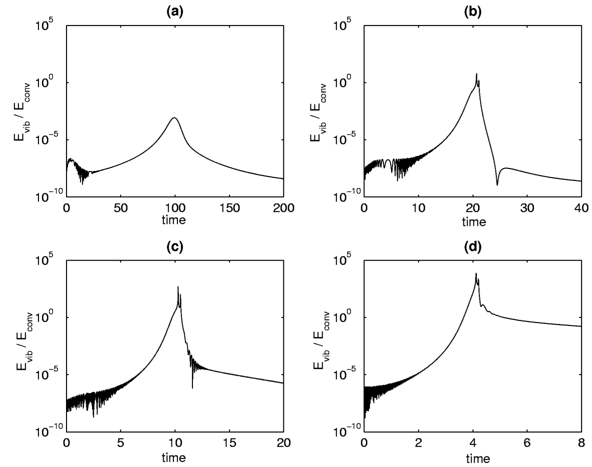

Figure 4. The ratio of the vibrational to the thermal energy (E_{vib}/E_{con}) vs time is shown at different velocity shear rates. Here $K_x = K_z = 10$, $K_y(0) = 200$ and $\sigma = -0.055$ ($\gamma = 0.95$). $R = 0.2, 1, 2, 5$ on the a, b, c and d panels, respectively. Initial values of perturbations are chosen to satisfy the condition: $E_{vib}(0)/E_{con}(0) \simeq 10^{-8}$. The ratio transiently picks and sometimes reaches values higher then 1 at $\tau = \tau^*$, where our definition of the mode energies is not accurate (see Fig. 3). Figure shows the remarkable growth of the vibrational energy compared to the convective one at $\tau = 2\tau^*$, where $K_y(2\tau^*)/K_x = -20$ and the separation of energies of acoustic oscillations and g-mode is highly accurate.

and the acoustic power is highly nonlinear.

Finally we note that in the present formalism we have focused on the waves with frequencies higher than the characteristic cut-off frequency for the acoustic waves in the convection zone. Shear flow initiates the qualitative change of the temporal variation scales of perturbations and the excitation of waves that are not trapped in the convective envelope. Hence, this mode conversion presents a new significant contribution into the channel of energy transfer from the dynamically active interior to the atmosphere of the Sun.

REFERENCES

Balmforth, N. J. 1992, Mon. Not. R. Astr. Soc., 255, 603.

Chagelishvili, G. D., Tevzadze, A. G., Bodo, G. and Moiseev, S. S., 1997, Phys. Rev. Letters 79, 3178.

Goldreich, P. and Kumar, P. 1990, Astrophys. J. 212, 243.

Lighthill, M. J., 1952, Proc. R. Soc. London Ser A 211, 564.

Rimmele, T. R., Goode, P. R., Harold, E., and Stebbins, R. T. 1995, Astrophys. J. 444, L119.

Ryu, D. and Goodman, J. 1992, Astrophys. J. 388, 438.

Stein, R. F., 1967, Solar Physics 2, 385.

Strous, L. H., Goode, P. R. and Rimmele, T. 2000, Astrophys. J. 535, 1000.